

## Analysis III for Engineering Students

### Work sheet 3

#### Exercise 1:

- a) Compute the Jacobian matrix of  $h$  using the chain rule:

$$h : \mathbb{R}^2 \xrightarrow{\mathbf{f}} \mathbb{R}^2 \xrightarrow{\mathbf{g}} \mathbb{R}$$
$$\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} u(x, y) \\ v(x, y) \end{pmatrix} \mapsto g(u, v).$$

- b) Compute the Jacobian matrix of  $f$  directly and also using the chain rule

$$\mathbf{f} : \mathbb{R}^2 \xrightarrow{\mathbf{f}_1} \mathbb{R}^3 \xrightarrow{\mathbf{f}_2} \mathbb{R}^2$$
$$\begin{pmatrix} r \\ s \end{pmatrix} \mapsto \begin{pmatrix} u = \sin(rs) \\ v = e^r + s \\ w = 1 - 2s^3 \end{pmatrix} \mapsto \begin{pmatrix} uw \\ vw \end{pmatrix}.$$

#### Exercise 2:

- a) Draw the following circles and ellipses

- (i)  $x^2 + y^2 = 5$ ,
- (ii)  $16x^2 + 25y^2 = 400$ ,
- (iii)  $x^2 + y^2 + 6(x - y) + 9 = 0$ ,
- (iv)  $x^2 + 2y^2 - 16y + 28 = 0$ .

Determine the  $(x, y)$  of the solution sets of the above equations using polar coordinates.

b) Draw the solution sets of the following areas in  $\mathbb{R}^3$

(i)  $x \leq 0, y \leq 0, 0 \leq z$  and  $x^2 + y^2 + z^2 \leq 9$

(ii)  $1 \leq z \leq 2, 0 \leq y$  and  $x^2 + y^2 \leq 9$

and represent them using cylindrical or spherical coordinates.

**Discussion:** 20.11. - 24.11.23