Department of Mathematics, University of Hamburg   WiSe 2023/24

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Analysis III
for Engineering Students
Homework sheet 3

Exercise 1:

Compute the derivative in direction $\boldsymbol{h} = (h_1, h_2)^T$ for the function $f : \mathbb{R}^2 \to \mathbb{R}$ $f(x, y) = x^2 + y$ at the point (x_0, y_0). What is the slope of the function at the point $(2, -3)$ in the directions given by the straight line $2x + 7y = 3$?

Exercise 2:

Given the coordinate transformation

$$\boldsymbol{\Phi}(r, \varphi) = \begin{pmatrix} x(r, \varphi) \\ y(r, \varphi) \end{pmatrix} = \begin{pmatrix} 2r \cos \varphi \\ 3r \sin \varphi \end{pmatrix}$$

with $(r, \varphi) \in Q :=]0, 1] \times \left] -\dfrac{\pi}{2}, \dfrac{\pi}{2} \right[$.

a) Compute $\boldsymbol{J} \, \boldsymbol{\Phi}(r, \varphi)$ and $\det(\boldsymbol{J} \, \boldsymbol{\Phi}(r, \varphi))$ as well as

b) $\boldsymbol{\Phi}^{-1}(x, y)$, $\boldsymbol{J} \, \boldsymbol{\Phi}^{-1}(x, y)$ and $\det(\boldsymbol{J} \, \boldsymbol{\Phi}^{-1}(x, y))$.

c) Make a sketch of Q and $\boldsymbol{\Phi}(Q)$.

Abgabetermin: 24.11.2023