

Analysis III for Engineering Students

Work sheet 1

Exercise 1:

Compute the gradients of the following functions $f : \mathbb{R}^2 \rightarrow \mathbb{R}$

- a) $f(x, y) = x^2 + 4y^2$, b) $f(x, y) = x^2 - 4y$, c) $f(x, y) = x^2 - 4y^2$,
d) $f(x, y) = x - 4y$

and draw a graph of different contour lines of functions in the area $[-2, 2] \times [-2, 2]$. These are the lines for which $f(x, y) = c$ with $c \in \mathbb{R}$ holds.

Exercise 2:

- a) Show that for a space variable x the solution to wave equation $u_{tt} = c^2 u_{xx}$ with a constant $c \in \mathbb{R}$ is given by the function

$$u(x, t) = 2 \sin(x + ct) + 3e^{x-ct}.$$

- b) Show that the function

$$u(x, y) = e^{-x} \sin y + (x + 5)(y - 6)$$

solves the Laplace equation $\Delta u = 0$.

Discussion: 23.10. - 27.10.23