Department of Mathematics, University of Hamburg **WiSe 2023/24**
Prof. Dr. J. Struckmeier
Dr. K. Rothe, T. Hassan

Analysis III for Engineering Students

Homework sheet 1

Exercise 1:

Given a function $f : \mathbb{R}^2 \to \mathbb{R}$ with $f(x, y) = 5x^2 - 3y^2$.

a) Compute all partial derivatives of f up to the 3rd order.

b) Visualize the graph of f in the area $[-3, 3] \times [-4, 4]$.

c) A tangent plane to the graph of a differentiable function f at the point $(x_0, y_0) \in D \subset \mathbb{R}^2$ is given by

$$z = z(x, y) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).$$

Compute the tangent plane for the given function f at the point $(x_0, y_0) = (3, -4)$.

d) Give a parametric representation of the contour line of f that goes through the point $(3, -4)$.

e) Compute the angle α between $\operatorname{grad} f(3, -4)$ and the tangential direction of the contour line of f at point $(3, -4)$.

Exercise 2:

Given a function $f : \mathbb{R}^2 \to \mathbb{R}$ with

$$f(x, y) = \begin{cases} \dfrac{xy^3}{x^4 + y^4} & \text{, if } \quad (x, y) \neq (0, 0) \\ 0 & \text{, if } \quad (x, y) = (0, 0) \, . \end{cases}$$

a) Check if f is continuous at the origin.

b) Visualize the graph of f over the parameter domain $[-1, 1] \times [-1, 1]$.

c) Compute the first partial derivatives of f and

d) check whether they are continuous at the origin.

Submission deadline: 27.10.23