Department of Mathematics
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WiSe 2023/24

Mathematics III Exam
(Module: Analysis III)

4. March 2024

Please mark each page with your name and your matriculation number.

Please write your surname, first name and matriculation number in **BLOCK CAPITALS** each in the following designated fields. These entries will be stored.

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I was instructed about the fact that the exam performance will only be assessed if the TUHH central examination office verifies my official admission before the exam's beginning.

(Signature)

Task no.	**Points**	**Examiner**
1		
2		
3		
4		
5		

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Exercise 1: (2+2 Points)

Given a function f with

$$f(x,y) = x^2 - \cos(y)e^{x-1}.$$

a) Compute

(i) the gradient and

(ii) the Hessian matrix.

b) Determine the 2nd degree Taylor polynomial for the function f at the point $(x_0, y_0) = (1, 0)$.

Exercise 2: (1+3+1 points)

Compute the extreme values of the function

$$f : \mathbb{R}^2 \to \mathbb{R} \quad \text{with} \quad f(x,y) = x^2 + y^2$$

under the constraint $g(x,y) = 9x^2 + 4y^2 - 36 = 0$ using the method of Lagrange multipliers:

a) Check the regularity condition for g.

b) Compute the candidates for extrema using the method of Lagrange multipliers.

c) Classify their type.

Exercise 3: (3 points)

Compute for the vector field

$$\boldsymbol{f} : \mathbb{R}^2 \to \mathbb{R}^2 \quad \text{with} \quad \boldsymbol{f}(x,y) = \begin{pmatrix} -xy \\ y^2 \end{pmatrix}$$

the line integral $\int_{\boldsymbol{c}} \boldsymbol{f}(\boldsymbol{x})d\boldsymbol{x}$, where the curve $\boldsymbol{c}$ runs through the left half circle

$$H := \left\{(x,y)^T \in \mathbb{R}^2 \mid x^2 + y^2 = 9,\ x \leq 0\right\}$$

in a mathematically positive orientation.

Exercise 4: (1+3 points)

Given a vector field $\boldsymbol{f} : \mathbb{R}^3 \to \mathbb{R}^3$ with

$$\boldsymbol{f}(x, y, z) = \begin{pmatrix} e^z \\ 4z - 2\sin(y) \\ xe^z + 4y + 1 \end{pmatrix},$$

a) Prove the existence of a potential for $\boldsymbol{f}$ without calculating it.

b) Compute a potential for $\boldsymbol{f}$.

Exercise 5: (2+2 points)

a) Make a sketch of the area K bounded by $0 \le y$, $0 \le z$ and $x^2 + y^2 + z^2 = 9$, and represent it using spherical coordinates.

b) Compute the mass of K with the density function $\rho = 8z + 3$ using spherical coordinates.