Mathematics III Exam
(Module: Analysis III)
26. August 2024

Please mark each page with your name and your matriculation number.

Please write your surname, first name and matriculation number in **BLOCK CAPITALS** each in the following designated fields. These entries will be stored.

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I was instructed about the fact that the exam performance will only be assessed if the TUHH central examination office verifies my official admission before the exam's beginning.

(Signature)

Task no.	Points	Examiner
1		
2		
3		
4		

$$\sum\ =$$

Exercise 1:   (5 points)

Compute all stationary points of the following function and determine their types

$$f(x, y) = x^3 - 3x + y^3 - 12y.$$

Exercise 2:   (1+1+3 points)

Given an implicit representation of a curve

$$f(x, y) := 4x^2 + 9y^2 - 36y = 0$$

a) determine the symmetries of the curve.

b) Compute the gradient of f.

c) Compute the points of curve with horizontal and vertical tangent.

Exercise 3: (2+2 points)

a) Make a sketch of the area Z enclosed by $0 \le z \le 5$ and $x^2 + y^2 \le 4$, and give its representation in cylindrical coordinates.

b) Given density $\rho(x, y, z) = 2z + 1$ compute the moment of inertia of Z about z-axis using cylindrical coordinates.

Exercise 4:　(1+1+3+1 points)

Given a vector field　$\boldsymbol{f}(x, y, z) = (0, 0, z)^T$　and a body

$$K = \left\{ (x, y, z)^T \in \mathbb{R}^3 \ \middle| \ x^2 + y^2 + z^2 \leq 25 \right\},$$

a) make a sketch of K.

b) For the surface S of the body K give a parameterization.

c) Calculate the flow(flux) of $\boldsymbol{f}$ through the surface S using parameterization from b).

d) Compute the volume integral　$\displaystyle\int_K \operatorname{div} \boldsymbol{f}(x, y, z)\, d(x, y, z)$.