Analysis III for Engineering Students
Work Sheet 5

Exercise 1:

Examine the implicitly given by the level set curve(s)

$$f(x, y) := y^4 - 2y^2 + x^4 - 2x^2 = 0.$$

In particular, determine

   a) the symmetries of the curve(s),

   b) the points of the curve with the horizontal and

   c) vertical tangents,

   d) the singular points of the curve and classify them,

   e) draw the level set.

Exercise 2:

Compute and classify the extrema of the function $f : \mathbb{R}^2 \to \mathbb{R}$ with $f(x, y) = 4x^2 + y^2$ on the circle $x^2 + y^2 - 2x = 3$

   a) by using Lagrange multipliers method and

   b) by using polar coordinate parameterization $\mathbf{c}$ of the circle and then solving the extreme problem $h(t) := f(\mathbf{c}(t))$.

Discussion: 19.12. - 23.12.2022