

Analysis III for Engineering Students

Work sheet 3

Exercise 1:

- a) Compute the Jacobian matrix of h using the chain rule:

$$h : \mathbb{R}^2 \xrightarrow{\mathbf{f}} \mathbb{R}^2 \xrightarrow{g} \mathbb{R}$$
$$\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} u(x, y) \\ v(x, y) \end{pmatrix} \mapsto g(u, v).$$

- b) Compute the Jacobian matrix of f directly and also using the chain rule

$$\mathbf{f} : \mathbb{R}^2 \xrightarrow{\mathbf{f}_1} \mathbb{R}^3 \xrightarrow{\mathbf{f}_2} \mathbb{R}^2$$
$$\begin{pmatrix} r \\ s \end{pmatrix} \mapsto \begin{pmatrix} u = \sin(rs) \\ v = e^r + s \\ w = 1 - 2s^3 \end{pmatrix} \mapsto \begin{pmatrix} uw \\ vw \end{pmatrix}.$$

Exercise 2:

- a) Draw the following circles and ellipses

- (i) $x^2 + y^2 = 5$,
- (ii) $16x^2 + 25y^2 = 400$,
- (iii) $x^2 + y^2 + 6(x - y) + 9 = 0$,
- (iv) $x^2 + 2y^2 - 16y + 28 = 0$.

Determine the (x, y) of the solution sets of the above equations using polar coordinates.

b) Draw the solution sets of the following areas in $\mathbb{R}^3$

(i) $x \leq 0, y \leq 0, 0 \leq z$ and $x^2 + y^2 + z^2 \leq 9$

(ii) $1 \leq z \leq 2, 0 \leq y$ and $x^2 + y^2 \leq 9$

and represent them using cylindrical or spherical coordinates.

Discussion: 21.11. - 25.11.22