

Analysis III for Engineering Students

Homework sheet 2

Exercise 1:

Compute the divergence and rate of rotation (curl) for the following vector fields with $x, y, z \in \mathbb{R}$

- a) $\mathbf{f}(x, y) = (\sin x \cos y, (x + y)^2)^T$,
- b) $\mathbf{g}(x, y) = (\sin y \cos x, -2xy)^T$,
- c) $\mathbf{f}(x, y) + \mathbf{g}(x, y)$,
- d) $\mathbf{h}(x, y, z) = (e^{x+y+z}, e^{x+y+z}, e^{x+y+z})^T$,
- e) $\mathbf{u}(x, y, z) = (yz, xz, xy)^T$,
- f) $2\mathbf{h}(x, y, z) - \mathbf{u}(x, y, z)$.

Exercise 2:

Given a function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ with

$$f(x, y) = \begin{cases} \frac{x^2 y}{2x^2 + 3y^2} & , \text{ if } (x, y) \neq (0, 0) \\ 0 & , \text{ if } (x, y) = (0, 0) . \end{cases}$$

- a) Draw the function in the domain $[-1, 1] \times [-1, 1]$.
- b) Compute all directional derivatives of f at the point $(x_0, y_0) = (0, 0)$.
- c) Check if f is (completely) differentiable at the point $(x_0, y_0) = (0, 0)$.