Department of Mathematics
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WiSe 2022/23

Mathematics III Exam
(Module: Analysis III)
6. March 2023

Please mark each page with your name and your matriculation number.

Please write your surname, first name and matriculation number in **BLOCK CAPITALS** each in the following designated fields. These entries will be stored.

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I was instructed about the fact that the exam performance will only be assessed if the TUHH central examination office verifies my official admission before the exam's beginning.

(Signature)

Task no.	Points	Examiner
1		
2		
3		
4		
5		

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Exercise 1: (2+2 Points)

Given a function f with

$$f(x,y) = y\cos(x) + xe^{y+1}$$

a) compute

  (i) the gradient and

  (ii) the Hessian matrix.

b) Determine the 2nd degree Taylor polynomial for the function f about a point $(x_0, y_0) = (0, -1)$.

Exercise 2: (1+3+1 points)

Compute the extrema of the function

$$f : \mathbb{R}^2 \to \mathbb{R} \quad \text{with} \quad f(x, y) = 2y^2 - 1$$

under the constraint $g(x, y) = x^2 + 4y^2 - 4 = 0$ using method of Lagrange multipliers:

a) Check the regularity condition for g.

b) Compute the candidates for extrema using method of Lagrange multipliers and

c) determine their type.

Exercise 3: (3 points)

Compute for the vector field

$$\boldsymbol{f} : \mathbb{R}^2 \to \mathbb{R}^2 \quad \text{with} \quad \boldsymbol{f}(x, y) = \begin{pmatrix} -x \\ y \end{pmatrix}$$

a line integral $\int_{\boldsymbol{c}} \boldsymbol{f}(\boldsymbol{x}) d\boldsymbol{x}$. The curve $\boldsymbol{c}$ runs through the upper quarter circle

$$H := \left\{ (x, y)^T \in \mathbb{R}^2 \mid x^2 + y^2 = 25,\ 0 \le x,\ 0 \le y \right\}$$

in a mathematically positive direction.

Exercise 4: (1+3 points)

Given a vector field $\boldsymbol{f} : \mathbb{R}^3 \to \mathbb{R}^3$ with

$$\boldsymbol{f}(x,y,z) = \begin{pmatrix} ye^x + 2xy + 3\cos(x) \\ e^x - z^2\sin(y) + x^2 \\ 2z\cos(y) + 2 \end{pmatrix},$$

a) Prove the existence of the potential for $\boldsymbol{f}$ without calculating it.

b) Compute the potential for $\boldsymbol{f}$.

Exercise 5: (2+2 points)

a) Make a sketch of the area K bounded by $0 \leq z$ and $x^2 + y^2 + z^2 = 1$, and represent it using the spherical coordinates.

b) Compute the mass of K with the density function $\rho = 1 + x^2 + y^2 + z^2$ using spherical coordinates.