# Mathematics III Exam <br> (Module: Analysis III) 

## 5. September 2023

Please mark each page with your name and your matriculation number.

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I was instructed about the fact that the exam performance will only be assessed if the TUHH central examination office verifies my official admission before the exam's beginning.

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| Task no. | Points | Examiner |
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## Exercise 1: (4 points)

Compute all stationary points of the following function and determine their types

$$
f(x, y)=\frac{x^{4}}{4}-2 x^{2}+\frac{y^{2}}{2}+2 y
$$

Exercise 2: ( $1+1+3$ points)
Given an implicit representation of a curve

$$
f(x, y):=x^{2}-6 x+4 y^{2}+5=0
$$

a) check the symmetries of the curve.
b) Compute the gradient of $f$.
c) Compute the points of curve with horizontal and vertical tangent.

Exercise 3: (2+2 points)
a) Make a sketch of the area $Z$ enclosed by $1 \leq z \leq 2$ and $x^{2}+y^{2} \leq 9$, and give its representation in cylindrical coordinates.
b) Given density $\rho(x, y, z)=z^{2}$ compute the moment of inertia of $Z$ about $z$-axis using cylindrical coordinates.

Exercise 4: $\quad(1+2+3+1$ points $)$
Given a vector field $\boldsymbol{f}(x, y, z)=(0, y z, 0)^{T}$ and a body

$$
K=\left\{(x, y, z)^{T} \in \mathbb{R}^{3} \mid x^{2}+y^{2}+z^{2} \leq 4, z \leq 0\right\}
$$

a) make a sketch of $K$.
b) Give parameterizations for each of the surface segments bounding $K$.
c) Calculate the flow(flux) of $\boldsymbol{f}$ through these boundary segments.

Hint: It holds $\int \sin ^{2}(\varphi) d \varphi=\frac{1}{2}(\varphi-\sin (\varphi) \cos (\varphi))$.
d) Compute the volume integral $\int_{K} \operatorname{div} \boldsymbol{f}(x, y, z) d(x, y, z)$.

