SoSe 2023

# Mathematics III Exam (Module: Analysis III)

#### 5. September 2023

Please mark each page with your name and your matriculation number.

Please write your surname, first name and matriculation number in **BLOCK CAPI-TALS** each in the following designated fields. These entries will be stored.

Surn	ame:													
$\mathbf{First}$	name	:												
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Stg.:	AIW	BU	BV	CI CS	ET	EUT	GES	IN IIW	LUN	M MB	MTE MEC	B SB	VT	

I was instructed about the fact that the exam performance will only be assessed if the TUHH central examination office verifies my official admission before the exam's beginning.

(Signature)	

Task no.	Points	Examiner
1		
2		
3		
4		



## Exercise 1: (4 points)

Compute all stationary points of the following function and determine their types

$$f(x,y) = \frac{x^4}{4} - 2x^2 + \frac{y^2}{2} + 2y.$$

#### **Exercise 2:** (1+1+3 points)

Given an implicit representation of a curve

$$f(x,y) := x^2 - 6x + 4y^2 + 5 = 0,$$

- a) check the symmetries of the curve.
- b) Compute the gradient of f.
- c) Compute the points of curve with horizontal and vertical tangent.

#### **Exercise 3:** (2+2 points)

- a) Make a sketch of the area Z enclosed by  $1\leq z\leq 2$  and  $x^2+y^2\leq 9\,,$  and give its representation in cylindrical coordinates.
- b) Given density  $\rho(x, y, z) = z^2$  compute the moment of inertia of Z about z-axis using cylindrical coordinates.

### **Exercise 4:** (1+2+3+1 points)

Given a vector field  $\boldsymbol{f}(x, y, z) = (0, yz, 0)^T$  and a body

$$K = \left\{ (x, y, z)^T \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 \le 4 \,, \, z \le 0 \right\} \,,$$

- a) make a sketch of K.
- b) Give parameterizations for each of the surface segments bounding K.
- c) Calculate the flow(flux) of f through these boundary segments.

*Hint:* It holds 
$$\int \sin^2(\varphi) d\varphi = \frac{1}{2}(\varphi - \sin(\varphi)\cos(\varphi)).$$

d) Compute the volume integral  $\int_{K} \operatorname{div} \mathbf{f}(x, y, z) d(x, y, z)$ .