M U 12.6 1

gegeben $f: I \to \mathbb{R}$

<u>Taylor</u> $T_f(x_0, t) = \sum_{k=0}^{\infty} f^{(k)}(x_0) \frac{(t-x_0)^k}{k!}$

Fourier $F_f(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos k\omega t + b_k \sin k\omega t$

$\omega = \frac{2\pi}{T}$

T Periode

$$\boxed{= \sum_{k=-\infty}^{\infty} \gamma_k e^{ik\omega t}}$$

$$\gamma_k = \frac{1}{T} \int_0^T f(t) e^{-ik\omega t} \, dt$$

$$\begin{pmatrix} a_k \\ b_k \end{pmatrix} = \frac{1}{T} \int_0^T f(t) \begin{pmatrix} \cos k\omega t \\ \sin k\omega t \end{pmatrix} dt$$

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Bsp

$f(t) = \sin t \qquad T = 2\pi \;, \omega = 1$

$a_k = 0 \;, b_k = \delta_{1k}$

$F_f(t) = \sin t$

$f(t) = \sin t \cos t \qquad T = 2\pi$

$= \frac{1}{2} \sin 2t$

$F_f(t) = \frac{1}{2} \sin 2t \qquad a_k = 0 \quad k = 0, \ldots$

$b_k = \frac{1}{2} \delta_{2k}$

$f(t) \qquad F_f(t) = \sum_{k=-\infty}^{\infty} \gamma_k e^{ik\omega t}$

$$\gamma_k = \frac{1}{T} \int_0^T f(t) e^{-ik\omega t} \, dt$$

1) Lin $\alpha f + \beta g$

$$\gamma_k = \frac{1}{T} \int_0^T (\alpha f(t) + \beta g(t)) e^{-ik\omega t} = \alpha \gamma_k + \beta \delta_k$$

2) Konj.

$$\overline{f(t)}$$

$$\overline{F_f(t)} = \overline{\sum_{k=-\infty}^{\infty} \gamma_k e^{ik\omega t}} =$$

$$= \sum_{k=-\infty}^{\infty} \overline{\gamma_k}\, e^{-ik\omega t} = \left[k \to -\tilde{k} \right.$$

$$= \sum_{\tilde{k}=-\infty}^{\infty} \overline{\gamma_{-\tilde{k}}}\, e^{i\tilde{k}\omega t} =$$

$$= \sum_{k=-\infty}^{\infty} \underline{\underline{\overline{\gamma_{-k}}}}\, e^{ik\omega t}$$

3)

$$f(-t)$$

$$F_f(-t) = \sum_{k=-\infty}^{\infty} \gamma_k e^{ik\omega(-t)} =$$

$$= \sum_{k=-\infty}^{\infty} \gamma_k e^{i(-k)\omega t} = \left[k \to -\tilde{k} \right.$$

$$= \sum_{\tilde{k}=-\infty}^{\infty} \underline{\gamma_{-\tilde{k}}}\, e^{i\tilde{k}\omega t}$$

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4)

$$f(ct) \approx \sum_{k=-\infty}^{\infty} \gamma_k e^{ik\omega(ct)}$$

$$\approx \sum_{k=-\infty}^{\infty} \gamma_k e^{ik(c\omega)t}$$

5)

$$f(t+a)$$

$$F_f(t+a) = \sum_{k=-\infty}^{\infty} \gamma_k e^{ik\omega(t+a)}$$

$$= \sum_{k=-\infty}^{\infty} \left(\gamma_k e^{ik\omega a}\right) e^{ik\omega t}$$

$$e^{in\omega t} f(t)$$

$$\cancel{F_f(t)}$$

$$F_{fe^{in\omega t}}(t) = \sum_{k=-\infty}^{\infty} \gamma_k e^{in\omega t} e^{ik\omega t}$$

$$= \sum_{k=-\infty}^{\infty} \gamma_k e^{i(k+n)\omega t} = \left[\tilde{k} = k+n\right]$$

$$= \sum_{\tilde{k}=-\infty}^{\infty} \gamma_{\tilde{k}-n} e^{i\tilde{k}\omega t}$$

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f

f'

$$F_f(t)' = \sum_{k=-\infty}^{\infty} (\gamma_k\, ik\omega)\, e^{ik\omega t}$$

$$F_f(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos k\omega t + b_k \sin k\omega t$$

$$\int_0^t F_f(\tau)\,d\tau = \frac{a_0}{2}t + \sum_{k=1}^{\infty} a_k \underbrace{\int_0^t \cos k\omega\tau\,d\tau} + b_k \underbrace{\int_0^t \sin k\omega\tau\,d\tau} = (*)$$

$$\int_0^t \cos k\omega\tau\,d\tau = \frac{\sin k\omega\tau}{k\omega}\Big|_0^t = \frac{\sin k\omega t}{k\omega}$$

$$\int_0^t \sin k\omega\tau\,d\tau = -\frac{\cos k\omega\tau}{k\omega}\Big|_0^t = -\frac{\cos k\omega t}{k\omega} + \frac{1}{k\omega}$$

$$(*) = \frac{a_0}{2}t + \underbrace{\sum_{k=1}^{\infty} \frac{b_k}{k\omega}}_{?} + \sum_{k=1}^{\infty} a_k \frac{\sin k\omega t}{k\omega} - b_k \frac{\cos k\omega t}{k\omega}$$

$$\int_0^t \tau F_f(\tau)\,d\tau = \int_0^t \left[\frac{a_0}{2}\tau + \sum_{k=1}^{\infty} a_k \tau \cos k\omega\tau + b_k \tau \sin k\omega\tau\right] d\tau$$

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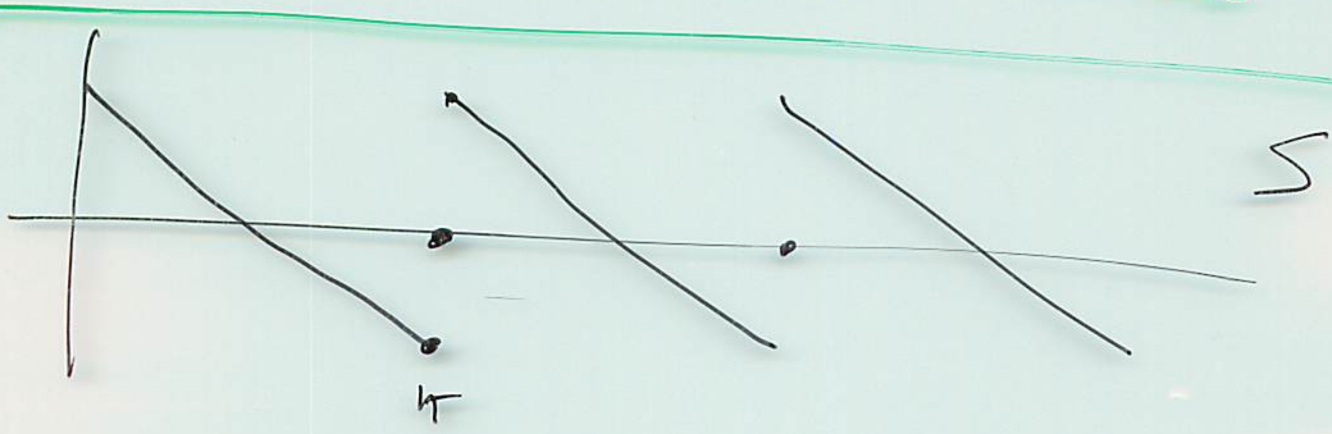
$$= \frac{a_0}{2}\frac{t^2}{2} + \sum_{k=1}^{\infty} a_k \underbrace{\int_0^t \tau \cos k\omega\tau \, d\tau} + b_k \underbrace{\int_0^t \tau \sin k\omega\tau \, d\tau}$$

$$\int_0^t \tau \sin k\omega\tau \, d\tau = -\tau \frac{\cos k\omega\tau}{k\omega}\Big|_0^t + \int_0^t \frac{\cos k\omega\tau}{k\omega} \, d\tau$$

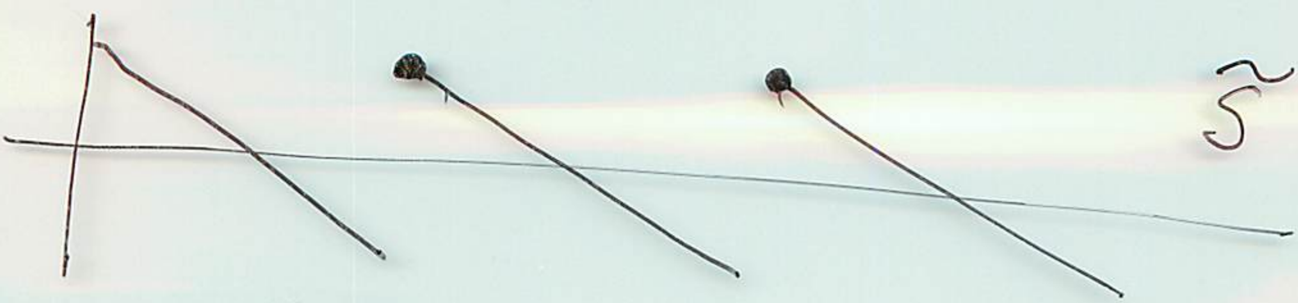
$$= -t\frac{\cos k\omega t}{k\omega} + \cancel{\frac{1}{k\omega}} + \int \cdots$$

$t = T$

$$= -\frac{T}{k\omega} + \underbrace{\int_0^T \cos k\omega\tau \, d\tau}_{=0}$$



$$F_f(\pi) = \frac{\pi - \pi}{2} = 0$$



12Vo ⑥

$F_{\tilde{S}}$ - - - - ——

$\tilde{a}_k = 0$

$\tilde{b}_k = \frac{2}{T}\int_0^T \tilde{S}(\tau) \sin k\omega t \, d\tau$

$= b_k$

$F_{\tilde{S}}(t) = F_S(t)$

$F_{\tilde{S}}(\pi) = 0 \neq \tilde{S}(\pi)$

$1 + 2\cos t + 2\cos 2t + \dots + 2\cos nt =$

$= 1 + e^{it} + e^{-it} + e^{2it} + e^{-2it} + \dots + e^{nit} + e^{-int} =$

$= 1 + \underbrace{(e^{it})^1}_{z} + (e^{it})^2 + \dots (e^{it})^n$

$+ (e^{-it})^1 + (e^{-it})^2 + \dots (e^{-it})^n$

$= \left[1 + z + z^2 + \dots z^n = \frac{z^{n+1} - 1}{z - 1} \right]$

$= \dots \dots =$

12 V6 Ct

$$\langle u,v\rangle = \frac{1}{T}\int_0^T \overline{u(t)}\, v(t)\, dt$$

$$\|u\| = \sqrt{\langle u,u\rangle} = \sqrt{\frac{1}{T}\int_0^T |u(t)|^2\, dt}$$

$$S_n = \frac{a_0}{2} + \sum_{k=1}^{n} a_k \cos k\omega t + b_k \sin k\omega t$$

$$\overline{S_n(t)}\, S_n(t) = \left(\overline{\qquad\qquad}\right)\left(\qquad\qquad\right)$$

$$\langle S_n, S_n\rangle = \frac{1}{T}\int_0^T \left(\overline{\sum_{k=1}^{n}\qquad}\right)\left(\sum_{k=1}^{n}\qquad\right) dt$$

$$= \frac{1}{T}\int_0^T \left(\frac{\overline{a_0}\,a_0}{4} + \ldots \quad \overline{\cos k\omega t \cdot \sin l\omega t} \right.$$

$$\cos k\omega t \cdot \cos l\omega t$$

$$\sin k\omega t \quad \sin l\omega t$$

$$= \frac{|a_0|^2}{4} \sum_{k=1}^{n} |a_k|^2 + |b_k|^2$$