$c:[a,b]\to\mathbb{R}^n \qquad c=c(t)$

$t=t(\tau) \qquad t:[\alpha,\beta]\to[a,b]$

bijektiv, monoton

$\tilde{c}(\tau)=c(t(\tau)) \qquad (\ )'=\frac{d}{d\tau}$

$\tilde{c}'(\tau)=\dot{c}(t(\tau))\cdot t'(\tau) \qquad (\ )^{\cdot}=\frac{d}{dt}$

$$\boxed{\int_{\tilde{c}} f(x)\,ds} := \int_\alpha^\beta f(\tilde{c}(\tau))\,\|\tilde{c}'(\tau)\|\,d\tau =$$

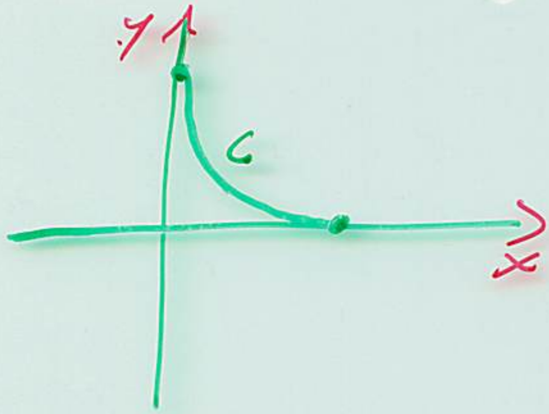
$$=\int_\alpha^\beta f(c(t(\tau)))\,\|\dot{c}(t(\tau))\,\underbrace{t'(\tau)}\|\,d\tau =$$

$$=\int_\alpha^\beta f(c(t(\tau)))\,\|\dot{c}(t(\tau))\|\,\underline{t'(\tau)\,d\tau}$$

$$=\int_a^b f(c(t))\,\|\dot{c}(t)\|\,dt$$

$$=:\boxed{\int_c f(x)\,ds}$$

MVo. 2



$$c(t) = \begin{pmatrix} \cos^3 t \\ \sin^3 t \end{pmatrix} \qquad t \in [0, \tfrac{\pi}{2}]$$

$$\|\dot c(t)\| = 3 \sin t \cos t$$

Massenverteilung $\rho = \rho(x) = x$

Gesamtmasse $M = \int_C \rho(x)\, ds :=$

$$= \int_0^{\frac{\pi}{2}} \rho(c(t))\, \|\dot c(t)\|\, dt = $$

$$= \int_0^{\frac{\pi}{2}} \rho(\cos^3 t)\, 3 \cos t \sin t \, dt$$

$$= 3 \int_0^{\frac{\pi}{2}} \cos^4 t \sin t \, dt = \frac{3}{5} \cdot \cos^5 t \,(-1) \Big|_0^{\frac{\pi}{2}}$$

$$\boxed{= \frac{3}{5}}$$

Schwerpunkt $\begin{pmatrix} x_S \\ y_S \end{pmatrix} = \frac{1}{M} \int_C \begin{pmatrix} x \\ y \end{pmatrix} \rho(x)\, ds =$

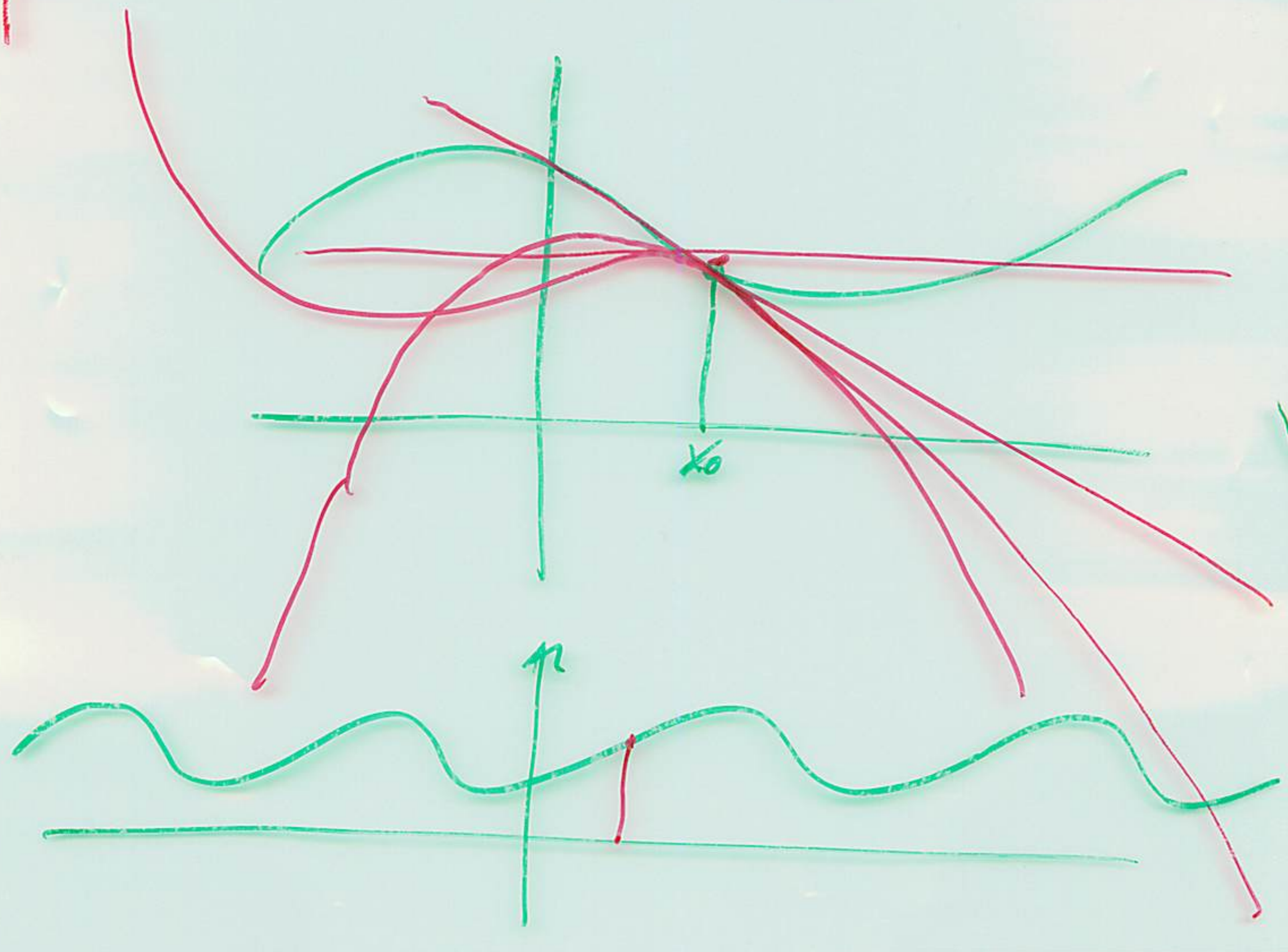
$$= \frac{1}{M} \int_C c(t)\, \rho(c(t))\, \|\dot c(t)\|\, dt =$$

$$= \frac{5}{3}\int_0^{\frac{\pi}{2}} \begin{pmatrix}\cos^3 t\\ \sin^3 t\end{pmatrix} \cos^3 t \; 3\cos t \sin t \, dt$$

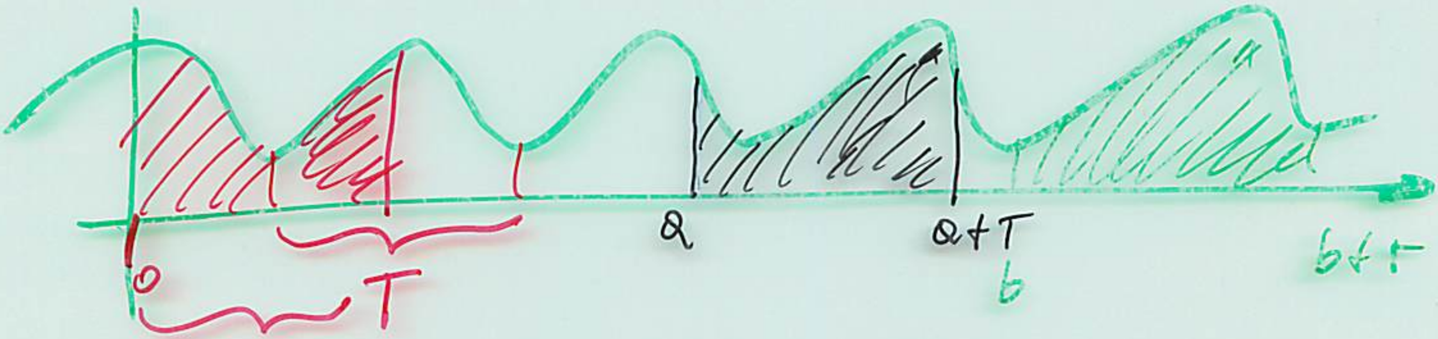
$$= \cancel{\frac{5}{3}} \int_0^{\frac{\pi}{2}} \begin{pmatrix}\cos^7 t \sin t\\ \cos^4 t \sin^4 t\end{pmatrix} dt = \frac{5}{3}\begin{pmatrix}\frac{3}{8}\\ ?\end{pmatrix} = \begin{pmatrix}\frac{5}{8}\\ ?\end{pmatrix}$$

Taylorreihe

f FRH $\Longrightarrow T_f(x_0, x) = \sum_{i=0}^{\infty} \frac{f^{(i)}(x_0)}{i!}(x-x_0)^i$

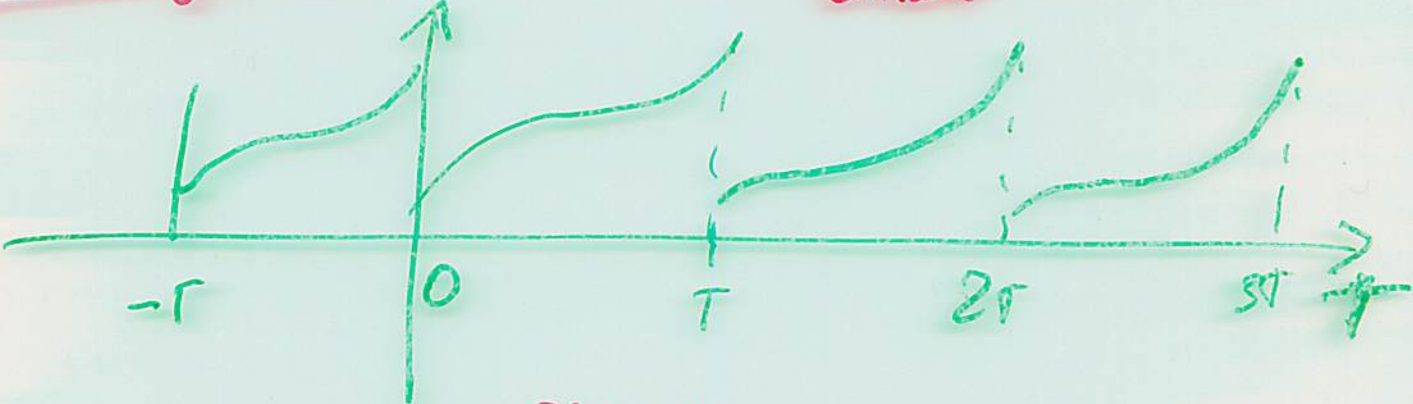


MV. 5

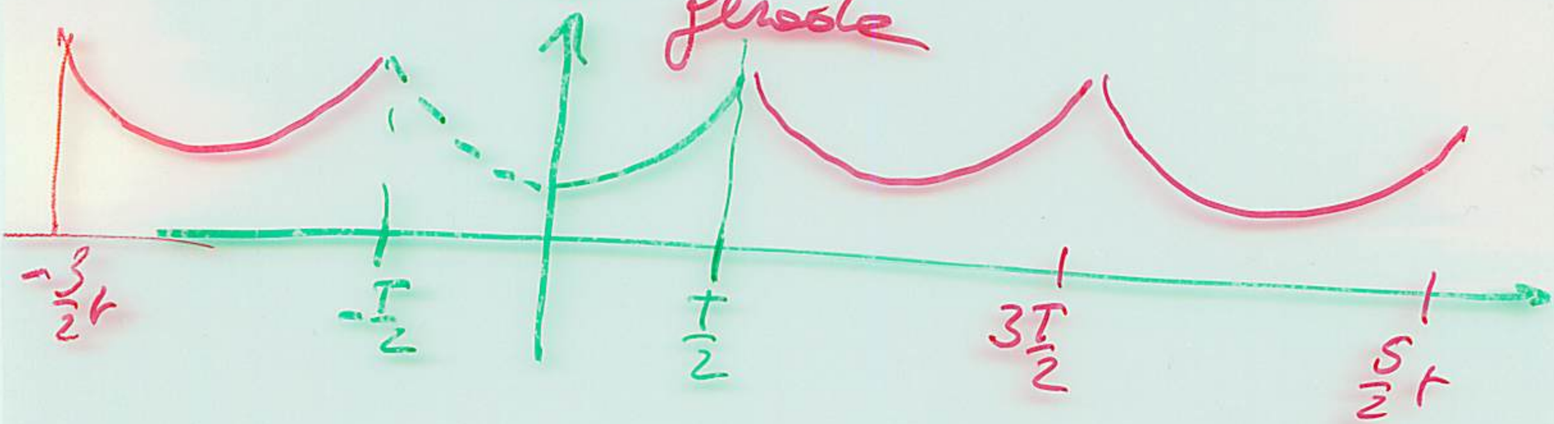


Fortsetzung

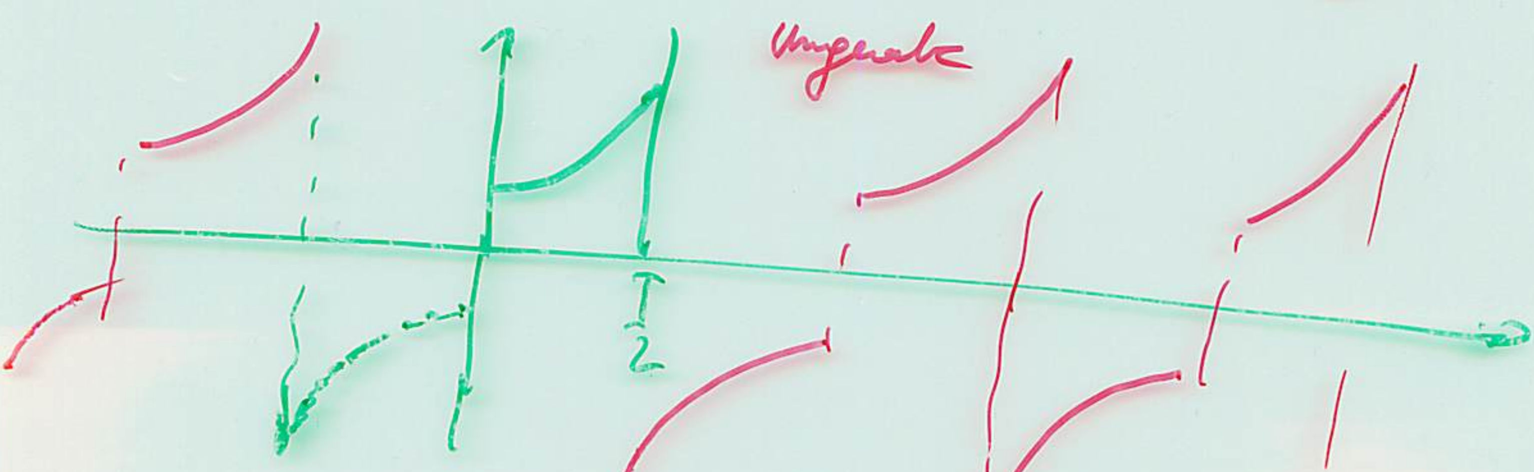
direkt

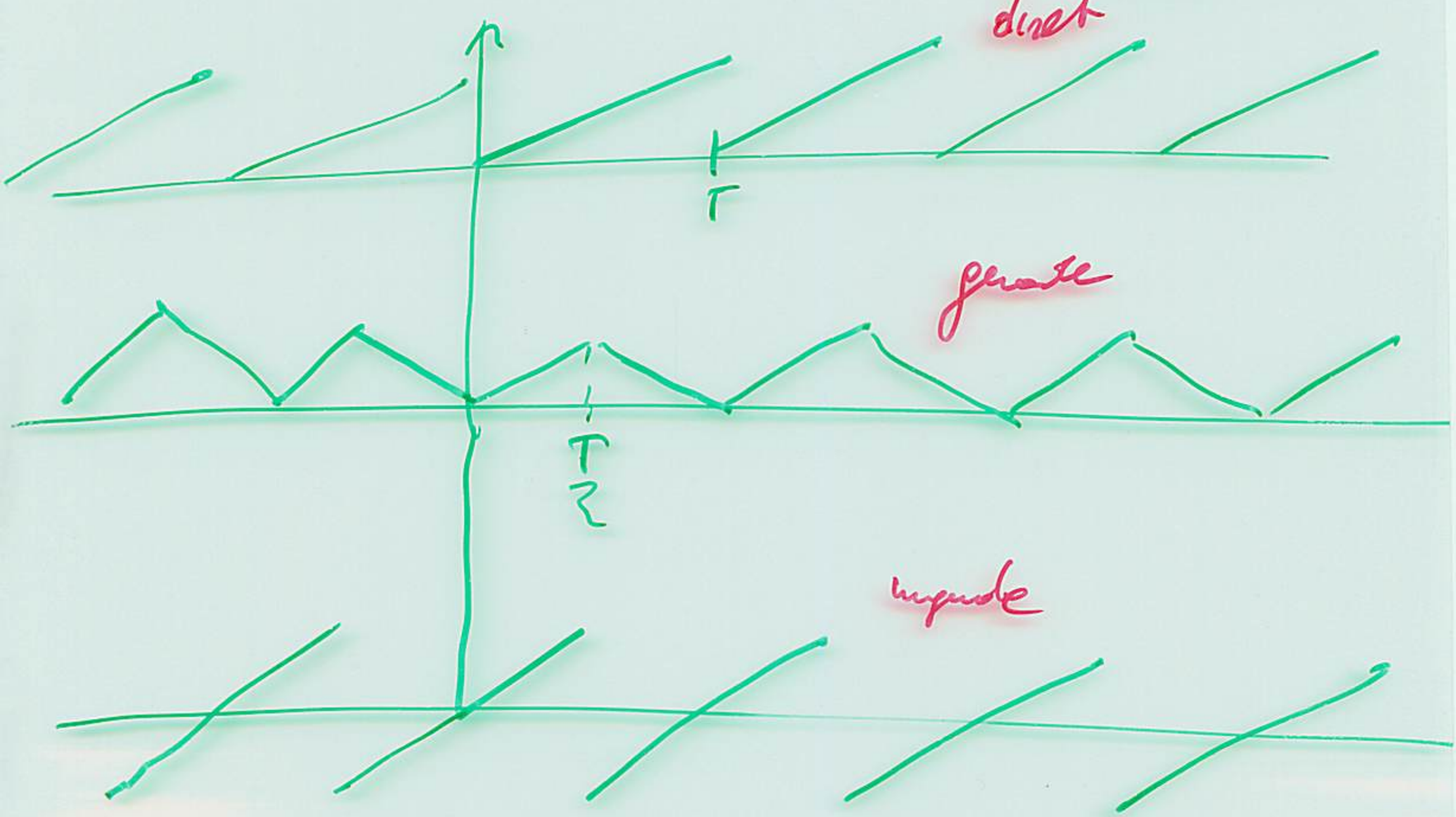


gerade



ungerade





$$\frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos(k\omega t) + b_k \sin(k\omega t) =$$

$$= \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \frac{e^{ik\omega t} + e^{-ik\omega t}}{2} + b_k \frac{e^{ik\omega t} - e^{-ik\omega t}}{2i}$$

$$\frac{1}{i} = -i$$

$$= \frac{a_0}{2} + \sum_{k=1}^{\infty} \left(\frac{a_k}{2} - i\frac{b_k}{2}\right) e^{ik\omega t} + \left(\frac{a_k}{2} + i\frac{b_k}{2}\right) e^{-ik\omega t}$$

$$= \sum_{k=-\infty}^{\infty} \gamma_k e^{ik\omega t}$$

$$\gamma_0 = \frac{a_0}{2} \qquad \gamma_k = \frac{1}{2}(a_k - i b_k) \quad , \gamma_{-k} = \frac{1}{2}(a_k + i b_k)$$

MVo 6

$a_0 = 2\gamma_0 \quad a_k = \gamma_k + \gamma_{-k} \quad b_k = \frac{\gamma_k - \gamma_{-k}}{i}$

ONS

$$\left\{ \dots, e^{-3i\omega t}, e^{-2i\omega t}, e^{-i\omega t}, 1, e^{i\omega t}, e^{2i\omega t}, \dots \right\}$$

$$u_k(t) = e^{ik\omega t}$$

$$\langle u_k, u_l \rangle = \frac{1}{T}\int_0^T \overline{u_k(t)}\, u_l(t)\, dt =$$

$$= \frac{1}{T}\int_0^T e^{-ik\omega t} e^{il\omega t}\, dt =$$

$$= \frac{1}{T}\int_0^T e^{i(l-k)\omega t}\, dt =$$

$$= \begin{cases} \frac{T}{T} = 1 & k = l \\ \frac{1}{T} \frac{e^{i(l-k)\omega t}}{i(l-k)\omega} \Big|_0^T = 0, & k \neq l \end{cases}$$

$\omega = \frac{2\pi}{T}$

$\{\cos k\omega t, \sin k\omega t \mid k \in \mathbb{Z}\}$ ONS

$\langle u,v \rangle \quad \frac{1}{T}\int_0^T u\,v\,dt$

$\frac{1}{T}\int_0^T \cos k\omega t \cos l\omega t\,dt =$

$= \frac{1}{T}\int_0^T \frac{1}{2}\left[\cos(k+l)\omega t + \cos(k-l)\omega t\right] dt$

$= \dots$

$\frac{1}{T}\int_0^T \sin k\omega t \sin l\omega t\,dt = \dots$

$\frac{1}{T}\int_0^T \sin k\omega t \cos k\omega t$

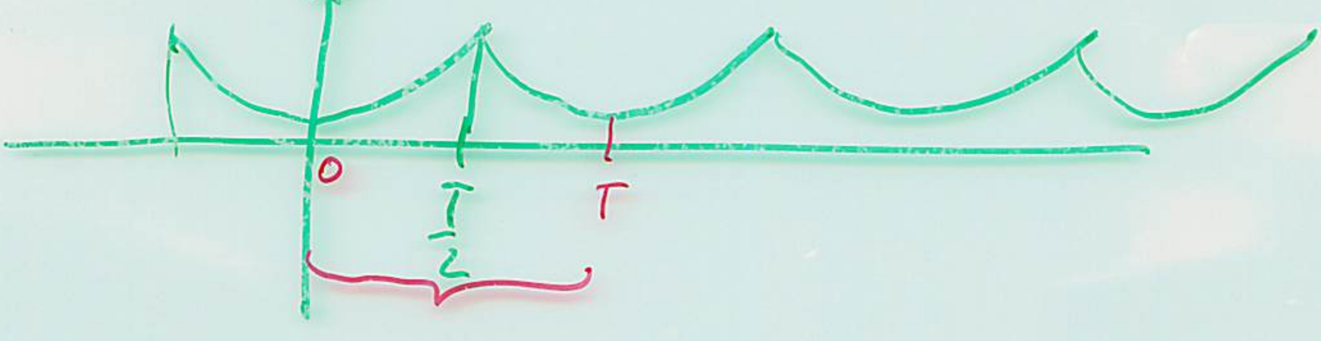
MVa

$$F(t) = \sum_{k=-\infty}^{\infty} \gamma_k e^{ik\omega t} \qquad \Big| \cdot \cancel{e^{-il\omega t}}\, e^{-il\omega t}$$

$$F(t) e^{-il\omega t} = \sum_{k=-\infty}^{\infty} \gamma_k e^{i(k-l)\omega t} \qquad \Big| \frac{1}{T}\int_0^T dt$$

$$\frac{1}{T}\int_0^T F(t) e^{-il\omega t}\, dt = \sum_{k=-\infty}^{\infty} \gamma_k \underbrace{\frac{1}{T}\int_0^T e^{i(k-l)\omega t}\, dt}_{\delta_{kl} = \begin{cases} 1 & k = l \\ 0 & k \neq l \end{cases}}$$

$$= \boxed{\gamma_l}$$



Mt

$$a_k = \frac{1}{T}\int_0^T f(t)\cos k\omega t\,dt$$

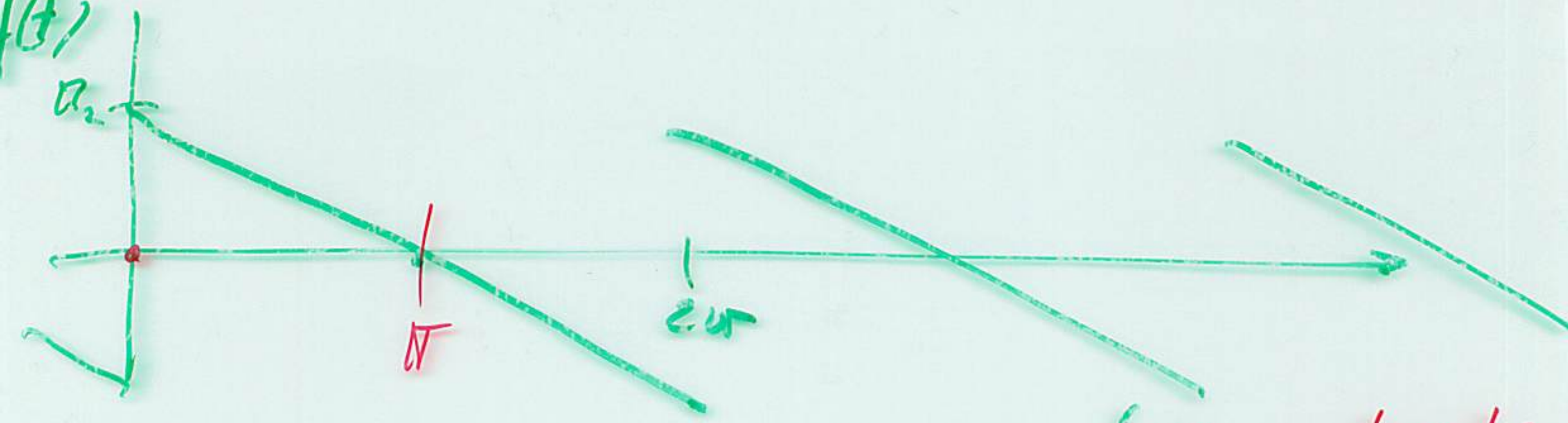
$$= \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\cos(k\omega t)\,dt$$

$$= \frac{1}{T}\int_{-\frac{T}{2}}^{0} f(t)\cos(k\omega t)\,dt + \int_0^{\frac{T}{2}}$$

$$= \left[\begin{array}{l} t \to -\tau \\ dt = -d\tau \end{array}\right]$$

$$= \frac{1}{T}\int_0^{\frac{T}{2}} \underbrace{f(-\tau)}_{=f(\tau)\ \underline{\text{gerade}}} \cos(k\omega\tau)\,d\tau + \frac{1}{T}\int_0^{\frac{T}{2}} f(t)\cos k\omega t\,dt$$

$$= \frac{2}{T}\int_0^{\frac{T}{2}} f(t)\cos\omega t\,dt$$



Ungerade

$$f(t) = \begin{cases} 0 & t=0,\ t=2\pi \\ \frac{1}{2}(\pi - t) & 0<t<2\pi \end{cases}$$

$\Rightarrow a_k \equiv 0$

$T = 2\pi$

$b_k = \frac{2}{\pi}\int_0^{\pi}$

$\Rightarrow \omega = \frac{2\pi}{T} = 1$

$\frac{T}{2} = \pi$

$$= \frac{4}{T}\int_0^{T/2} f(t)\sin k\omega t\, dt = \frac{2}{\pi}\int_0^{\pi}\frac{\pi - t}{2}\sin kt\, dt$$

$$= \dots \frac{1}{k}$$

$$\int \underset{u}{t}\, \underset{v'}{\sin kt} = -\frac{t\cos kt}{k} + \int 1 \cos kt$$