Balancing for Nonlinear Systems MOR

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Abstract

We review a nonlinear balancing approach presented in 2000. Unlike the method introduced by Scherpen (1994), which requires the solution of PDE's, the novel method is computationally efficient and works also for unstable systems.

First, we argue that a generalization for balancing nonlinear systems should be based upon three principles: 1) Balancing should be defined with respect to a nominal flow; 2) Only Gramians defined over small time intervals should be used in order to preserve the accuracy of the linear perturbation model and; 3) Linearization should commute with balancing, in the sense that the linearization of a globally balanced model should correspond to the balanced linearized model in the original coordinates.

The first two principles lead to local balancing, but it is shown that an integrability condition generically provides an obstruction towards a notion of a globally balanced realization in the strict sense. By relaxing the conditions of "strict balancing" in various ways useful system approximations may be obtained.

However, the information obtained by *local balancing* of a nonlinear system already provides a lot of useful information about the dominant dynamics of the system and the topology of the state space. To accomplish local balancing, two Riemannian metrics are specified: One models the local reachability properties and one models the local observability properties. In general these are incompatible, inducing a different global topology, and thus explaining the aforementioned obstruction. Locally, it still may be possible to match these up, and local balancing at a point P corresponds to bending and reshaping the manifolds without tearing so that near P there is a snug fit (osculating contact) between the induced manifolds. Unlike the linear case, sensitivity and reduced modeling must be local concepts, and lead at best to a hybrid reduced model with modes of *different dimension*.

Finally, model reduction invariably leads to the deletion of information regarding the original system. This loss of information is equivalent to the introduction of uncertainty. Yet such uncertainty is typically not modelled in classical approaches to model reduction. Therefore one should quantify this uncertainty and provide realistic uncertainty bounds on the dynamics of the reduced order model, while not adding significantly to the computational complexity. This *uncertainty equivalence* is based on a maximum likelihood estimate of the discarded state and stochastic system theory. It is shown that the original balanced realization provides enough information to specify such bounds.

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