Structure preserving model reduction of port-Hamiltonian systems

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1 Introduction

Port-based network modeling of (lumped-parameter) physical systems leads directly to their representation as port-Hamiltonian systems which are an important class of passive state-space systems. At the same time modeling of physical systems often leads to high-dimensional dy namical models. State-space dimensions are enormously high as well if distributed-parameter models are spatially discretized. Therefore an important issue concerns model reduction of these high-dimensional systems, both for analysis and control. The goal of this work is to show that the specific model reduction techniques of linear port-Hamiltonian systems preserve port-Hamiltonian structure, and, as a consequence, passivity.

2 Model reduction of uncontrollable/unobservable linear port-Hamiltonian systems

It is a well known fact that port-Hamiltonian systems are not only passive but also have a specific natural structure which depends on the total energy or so-called Hamiltonian. Other important issues like interconnection between port-Hamiltonian systems and energy dissipation are also reflected by the port-Hamiltonian structure. General theory on port-Hamiltonian systems can be found in [3]. We will show by applying Kalman decomposition that the reduction of the dynamics of an uncontrollable/unobservable linear port-Hamiltonian system to a dynamics on the reachability/observability subspace preserves the port-Hamiltonian structure. This result holds both for energy and co-energy variable representations of linear port-Hamiltonian systems and it is also shown that the reduced models in the co-energy variable representation take a somewhat ”dual” form to the reduced models obtained in the energy variable representation.

3 Model reduction of general linear port-Hamiltonian systems

Within the systems and control literature a popular and elegant tool for model reduction is balancing. One favorable property of model reduction based on balancing, as compared with other techniques such as modal analysis, is that the approximation of the dynamical system is explicitly based on its input-output properties. Balancing for lossless and passive systems is considered in [4]. Since standard open-loop balancing assumes that the system is asymptotically stable it cannot be directly applied to lossless port-Hamiltonian systems. In order to overcome this difficulty it is useful to switch to scattering representation. We will apply ”Effort constraint” and ”Flow constraint” methods of model reduction to linear port-Hamiltonian systems in scattering representation and show that the reduced-order models inherit properties of the port-Hamiltonian structure.

References

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