Parametric Model Order Reduction for ODEs as Reduction Method for Semi-Explicit Systems of DAEs

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Summary. Differential algebraic equations (DAEs) result in the mathematical modeling of a wide variety of problems (like electric circuit design). One rich source of DAEs in practice is obtained by limit of singular perturbed ordinary differential equations (ODEs), where a small parameter tends to zero. Model order reduction (MOR) has proven to be an efficient tool for dealing with computational complexity in the simulation of large interconnect electrical networks for the case of ODEs. We study a parametric MOR technique as Reduction Method for singular-perturbed semi-explicit systems to reduce the order of DAEs.

1 Introduction

For large systems of ODEs, efficient MOR methods already exist in the linear case. We want to generalise according techniques to the case of DAEs. On the one hand, a high-index DAE problem can be converted into a lower-index system by analytic differentiations, see [1]. A reduction to index zero yields an equivalent system of ODEs. On the other hand, a regularisation is directly feasible in case of semi-explicit systems of DAEs. Thereby, we obtain a singular perturbed problem of ODEs. Thus according MOR techniques can be applied to the ODE system. An MOR approach for DAEs is achieved by considering the limit to zero of the artificial parameter.

2 Regularization of semi-explicit systems of DAEs

We consider a semi-explicit system of DAEs:

$$\begin{aligned} \frac{\mathrm{d}y}{\mathrm{d}t} &= f(y(t), z(t)) \qquad y : \mathbb{R} \to \mathbb{R}^k \\ 0 &= g(y(t), z(t)) \qquad z : \mathbb{R} \to \mathbb{R}^l \end{aligned} \tag{1}$$

with differential and perturbation index 1 or 2. For the construction of numerical methods, the direct approach is based on the formulation:

$$\begin{aligned} \frac{\mathrm{d}y}{\mathrm{d}t} &= f(y(t), z(t)) \\ \epsilon\frac{\mathrm{d}z}{\mathrm{d}t} &= g(y(t), z(t)) \end{aligned} \quad\Leftrightarrow\quad \begin{aligned} \frac{\mathrm{d}y}{\mathrm{d}t} &= f(y(t), z(t)) \\ \frac{\mathrm{d}z}{\mathrm{d}t} &= \frac{1}{\epsilon}g(y(t), z(t)). \end{aligned} \tag{2}$$

Numerical schemes for ODEs can be applied to the system (2). The limit $\epsilon \to 0$ yields an approach for solving the DAE (1). The idea is to use MOR in case of large dimensions $n = k + l$ for DAE systems (1), where k as well as l may be large. Based on the system (2), MOR techniques for ODEs can be substituted. This approach represents a parametric MOR, since the artificial parameter ϵ is involved. Finally, the limit $\epsilon \to 0$ results in an MOR scheme for DAEs (1). The reduction can be carried out by means of different techniques [2].

Example. We consider a linear forced LC-oscillator, which includes one inductor, m capacitors and n resistors:

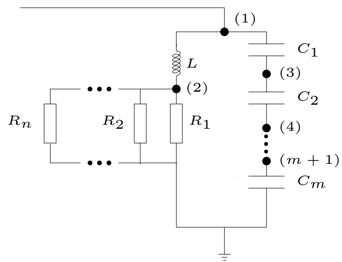

Fig. 1. Linear forced LC-oscillator.

Modified nodal analysis (MNA) as well as a sparse tableau modeling yield a corresponding system of semi-explicit DAEs. We apply reduction techniques based on the above approach.

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References

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