Parametric Model Order Reduction in MEMS and IC Design

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At our institute, model order reduction (MOR) methods have been used with great success in model generation and simulation of integrated circuits as well as heterogeneous systems such as micro electrical mechanical systems (MEMS), see [1], [6], [7], [7] for reference. In general, we use tools like ANSYS or Simlab's PCBMod to obtain linear time-invariant descriptor systems of the form:

$$\left.\begin{aligned} \mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} &= \mathbf{B}^{\text{in}}\mathbf{u} \\ \mathbf{y} &= \mathbf{B}_1^{\text{out}}\mathbf{q} + \mathbf{B}_2^{\text{out}}\dot{\mathbf{q}} + \mathbf{D}\mathbf{u} \end{aligned}\right\} \tag{1}$$

$\mathbf{M}, \mathbf{D}, \mathbf{K} \in \mathbb{R}^{N\times N}; \mathbf{B}^{\text{in}} \in \mathbb{R}^{N\times p}; \mathbf{B}_1^{\text{out}}, \mathbf{B}_2^{\text{out}} \in \mathbb{R}^{q\times N}; \mathbf{D} \in \mathbb{R}^{q\times p}$ are sparse, but with the state space dimension N reaching magnitudes of $10^5 \ldots 10^7$, time domain simulations become inefficient, especially when (1) is only part of a bigger system obtained by coupling.

During the last decades, Krylov subspace based MOR schemes proved as a practical way to efficiently reduce the state space dimension ([2], [4]). But we lately face a growing demand for an extension of these methods to parameter dependent models. Thus, Fraunhofer started CAROD (Computer-Aided Robust Design) [3], a joint research project for tools incorporating manufacturing tolerances during simulation and design.

As the tools for the generation of the system matrices of (1) behave like black boxes, the analytical relationship between parameter values and resulting system matrices are unknown. Thus, in out terms parametric model order reduction (PMOR) involves two aspects:

1. Derivation of a parameter dependent full order system from a series of systems of form (1) related to a series of fixed parameter values
2. Reduction of the state space dimension of the parameterized system while keeping the parameter dependency of the system matrices

The talk will outline the state of the art of parametric model order reduction at Fraunhofer IIS/EAS including real world examples, pitfalls and open problems.

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[2] Freund, R. W.: Model Reduction Methods Based on Krylov Subspaces; Numerical Analysis Manuscript No. 03-4-01, Bell Laboratories, 2003

[3] http://www.nusim.fraunhofer.de/nusim-contents/carod.html

[4] Köhler, Andreas: Modellreduktion von linearen Deskriptorsystemen erster und zweiter Ordnung mit Hilfe von Block-Krylov-Unterraumverfahren. Diplomarbeit, TU Bergakademie Freiberg, 2006.

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[7] Schwarz, Peter; Schneider, Peter: Model Library and Tool Support for MEMS Simulation. SPIE's conference "Microelectronic and MEMS Technology", Edinburgh, Scotland. 30.5.-1.6.2001, SPIE Proceedings Series Volume 4407