On recycling Krylov subspaces for solving linear systems with successive right-hand sides with applications in model reduction

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We discuss the numerical solution of successive linear systems of equations

$$Ax = b_i,\ i = 1, 2, \dots m, \tag{1}$$

by iterative methods based on recycling Krylov subspaces. We propose two recycling algorithms, which are both based on the generalized conjugate residual (GCR) method. The recycling methods reuse the descent vectors computed while solving the previous linear systems $Ax = b_j,\ j = 1, 2, \dots, i-1$, such that a lot of computational work can be saved when solving the current system $Ax = b_i$. When compared with standard GCR, the proposed algorithms are robust when they are applied to solve many successive linear systems. These linear systems arise, e.g., in model reduction methods for systems with many terminals.

When the sizes of the systems are very large, instead of direct methods like LU decomposition, iterative methods are preferred to solve the system. One simple way of solving systems in (1) is to solve each system one after another by standard iterative methods such as GCR, GMRES, etc.. Since the systems above share the same coefficient matrix A, we try to exploit the information derived from the previous system solutions to seep up the solution of the next system. With this motivation, and based on the standard GCR method, the recycling algorithms proposed in this paper reuse the descent vectors in GCR for the solution of the previous systems as the descent vectors for the solution of the next system. In this way, the computation of matrix-vector multiplications is saved at many iteration steps. The algorithms have the same convergence property as the standard GCR method has, that is: for each system, the approximate solution computed at each iteration step minimizes the residual in the affine subspace comprising the initial guess and the descent vectors.

Conventional block iterative methods (see [2] and its references) cannot be applied to (1), since they require that the right-hand sides are available simultaneously. A new method [1] is proposed to solve systems $A_i x_i = b_i$, which is the generalized form of (1), whereas it turns out to be quite involved for systems with same coefficient matrix A as in (1). The proposed algorithms are much simpler and easy to implement.

References

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