L^∞-error estimates in non-convex domains with application to optimal control

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Model problem

We discuss the optimal control problem

$$\begin{aligned} J(\bar{u}) &= \min J(u), \\ J(u) &:= F(Su, u), \\ F(y, u) &:= \frac{1}{2}\|y - y_d\|^2_{L^2(\Omega)} + \frac{\nu}{2}\|u\|^2_{L^2(\Omega)}, \end{aligned}$$

where the associated state $y = Su$ to the control u is the weak solution of the state equation

$$-\Delta y = u \quad \text{in } \Omega, \qquad y = 0 \quad \text{on } \Gamma = \partial\Omega,$$

and the control variable is constrained by

$$a \leq u(x) \leq b \qquad \text{for a.a. } x \in \Omega.$$

Ω is assumed to be two-dimensional and non-convex.

Results from literature for L^∞-estimates

Results in convex domains

- [Meyer/Rösch, 2004] discrete control, no postprocessing, 2D

$$\|\bar{u} - \bar{u}_h\|_{L^\infty(\Omega)} = O(h)$$

- [Hinze, 2004] semidiscrete approach

$$\|\bar{u} - \bar{u}_h\|_{L^\infty(\Omega)} \lesssim \|(S - S_h)y_d\|_{L^\infty(\Omega)} + \|(S^* - S_h^*)S\bar{u}\|_{L^\infty(\Omega)} + \begin{cases} h^2|\log h|^{1/2}\|\bar{u}\|_{L^2(\Omega)} & \text{(2D)} \\ h^{3/2}\|\bar{u}\|_{L^2(\Omega)} & \text{(3D)} \end{cases}$$

L^∞-error estimates for the state equation

State equation:

$$-\Delta y = f \quad \text{in } \Omega, \qquad y = 0 \quad \text{on } \Gamma = \partial\Omega,$$

Results in literature:

- [Frehse/Rannacher, '76]

$$\|y - y_h\|_{L^\infty(\Omega)} \le Ch^2 |\log h| |y|_{W^{2,\infty}(\Omega)}$$

- [Schatz/Wahlbin, '78/'79]

$$\|y - y_h\|_{L^\infty(\Omega)} \le Ch^{2-\epsilon}$$

for "smooth" right-hand side.

Aim:

- Estimate for non-convex domains.
- Seperate the constant from the norm of the function to be approximated.

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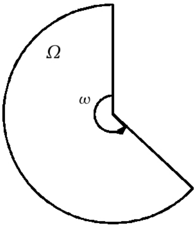
Corner singularities (2D)



The solution y of

$$-\Delta y = f \text{ in } \Omega, \qquad y = 0 \text{ on } \partial\Omega$$

is not contained in the Sobolev space $W^{2,2}(\Omega)$, if $\omega > \pi$.

Instead, one can write

$$y = y_r + y_s$$

where $y_r \in W^{2,2}(\Omega)$ and

$$y_s = \xi(r)\gamma r^\lambda \sin(\lambda\varphi) \quad \text{with } \lambda = \frac{\pi}{\omega}.$$

$\xi(r)$ is a smooth cut-off function and γ a coefficient.

Regularity

The solution can be described in weighted Sobolev spaces

$$V^{k,p}_\beta(\Omega) = \left\{ y \in \mathcal{D}' : \sum_{|\alpha|\le k} \left(\int_\Omega |r^{\beta-|\alpha|+k} D^\alpha y|^p \right)^{1/p} < \infty \right\}$$

$$V^{k,\infty}_\gamma(\Omega) = \left\{ y \in \mathcal{D}' : \sup_{\substack{x\in\Omega \\ |\alpha|\le k}} \left(|r^{\gamma-|\alpha|+k} D^\alpha y(x)| \right) < \infty \right\}.$$

One has the a priori estimates

$$\|y\|_{V^{2,2}_\beta(\Omega)} \lesssim \|f\|_{L^2(\Omega)} \quad \text{for } \beta > 1-\lambda,$$

$$\|y\|_{V^{2,\infty}_\gamma(\Omega)} \lesssim \|f\|_{C^{0,\sigma}(\Omega)} \quad \text{for } \gamma > 2-\lambda.$$

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Mesh grading

In order to get an optimal rate of convergence, mesh grading is necessary. We set the element size according to

$$h_T = \begin{cases} h^{1/\mu} & \text{if } r_T = 0 \\ h r^{1-\mu} & \text{if } r_T > 0 \end{cases}$$

where h is the global mesh parameter and r is the distance to the corner. One has to choose

$$\mu < \begin{cases} \lambda & \text{for optimal convergence rate in } L^2(\Omega). \\ \frac{\lambda}{2} & \text{for optimal convergence rate in } L^\infty(\Omega). \end{cases}$$

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Subsets of Ω

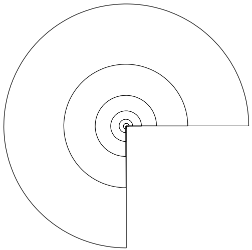
Divide Ω in different subsets

$$\Omega = \bigcup_{j=0}^{I} \Omega_j$$

where

$$\Omega_j = \{x : d_{j+1} \leq |x - v| \leq d_j\} \qquad j \neq I$$
$$\Omega_I = \{x : |x - v| \leq d_I\}$$

and $d_j = 2^{-j}$, $d_I \sim h^{2/\lambda}$.



Local error estimates

Lemma

For $J = I, I-1$ one has

$$\|y - y_h\|_{L^\infty(\Omega_J)} \lesssim |\log h|^{1/2} h^2 \|y\|_{V_\gamma^{2,\infty}(\Omega_J')} + |\log h|^{1/2} d_J^{-1} \|y - y_h\|_{L^2(\Omega_J')}.$$

For $J \neq I, I-1$ the inequality

$$\|y - y_h\|_{L^\infty(\Omega_J)} \lesssim |\log h| \min_{\chi \in V_h} \|y - \chi\|_{L^\infty(\Omega_J')} + d_J^{-1} \|y - y_h\|_{L^2(\Omega_J')}$$

is valid.

Lemma

The inequality

$$d_J^{-1} \|y - y_h\|_{L^2(\Omega_J')} \lesssim h^2 |\log h|^{1/2} \|y\|_{V_\gamma^{2,\infty}(\Omega)}$$

is valid.

L^∞-estimate for the state equation

Theorem

Let y be the solution of the boundary value problem with a right-hand side $f \in C^{0,\sigma}(\Omega)$. The finite element error can be estimated by

$$\|y - y_h\|_{L^\infty(\Omega)} \leq Ch^2 |\log h| \|f\|_{C^{0,\sigma}(\Omega)}$$

on finite element meshes with grading parameter $\mu < \lambda/2$.

A mesh is graded, if $\mu < 1$. This means, that mesh grading is necessary for all corners with

$$\omega > \frac{\pi}{2}.$$

Back to optimal control

$$\min J(u) = \frac{1}{2}\|Su - y_d\|_{L^2(\Omega)} + \frac{\nu}{2}\|u\|_{L^2(\Omega)}$$

subject to

$$-\Delta y = u \quad \text{in } \Omega, \qquad y = 0 \quad \text{on } \partial\Omega$$

and

$$a < u < b \quad \text{a. e. in } \Omega.$$

with $a < b$, $\nu > 0$, $\Omega \subset \mathbb{R}^2$ non-convex with one reentrant corner.

Optimality system

The optimal control $\bar{u}$ is the unique solution of the system

$$\begin{aligned} \bar{y} &= S\bar{u}, \\ \bar{p} &= S^*(\bar{y} - y_d), \\ \bar{u} &= \Pi_{[a,b]}\left(-\frac{1}{\nu}\bar{p}\right) \end{aligned}$$

with the projection

$$\Pi_{[a,b]}f(x) := \max(a, \min(b, f(x))).$$

The control is discretized by piecewise constant functions. State and adjoint state are approximated with piecewise linear functions. Denote by S_h the solution operator of the discrete system and by R_h the operator

$$R_h u(x) = u(S_T) \quad \text{if } x \in T, \quad S_T \text{ centroid of } T.$$

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Discretization of the OCP

Find $\bar{u}_h \in U_h^{ad}$ such that

$$J_h(\bar{u}_h) = \min_{u_h \in U_h^{ad}} \frac{1}{2}\|S_h u_h - y_d\|^2_{L^2(\Omega)} + \frac{\nu}{2}\|u_h\|^2_{L^2(\Omega)}$$

with

$$\begin{aligned} U_h &= \{u \in L^2(\Omega) : u|_T \in \mathcal{P}_0 \ \forall T \in T_h\} \\ U^{ad} &= \{u \in L^2(\Omega) : a \le u(x) \le b \text{ a.e. in } \Omega\} \\ U_h^{ad} &= U_h \cap U^{ad} \end{aligned}$$

Discrete optimality system:

$$\begin{aligned} \bar{y}_h &= S_h \bar{u}_h \\ \bar{p}_h &= S_h^*(\bar{y}_h - y_d) \\ \bar{u}_h &= \Pi_{U_h^{ad}}\left(-\frac{1}{\nu} R_h \bar{p}_h\right). \end{aligned}$$

L^∞-error estimate for OCP

Theorem

On a family of meshes with grading parameter $\mu < \frac{\lambda}{2}$ the estimates

$$\|\bar{y}_h - \bar{y}\|_{L^\infty(\Omega)} \le ch^2|\log h| \left(\|\bar{u}\|_{C^{0,\sigma}(\bar{\Omega})} + \|y_d\|_{C^{0,\sigma}(\bar{\Omega})}\right)$$

$$\|\bar{p}_h - \bar{p}\|_{L^\infty(\Omega)} \le ch^2|\log h| \left(\|\bar{u}\|_{C^{0,\sigma}(\bar{\Omega})} + \|y_d\|_{C^{0,\sigma}(\bar{\Omega})}\right)$$

are valid.

Proof:

$$\begin{aligned}\|\bar{y} - \bar{y}_h\|_{L^\infty(\Omega)} &= \|S\bar{u} - S_h\bar{u}_h\|_{L^\infty(\Omega)} \\ &\le \|S\bar{u} - S_h\bar{u}\|_{L^\infty(\Omega)} + \|S_h\bar{u} - S_hR_h\bar{u}\|_{L^\infty(\Omega)} + \|S_hR_h\bar{u} - S_h\bar{u}_h\|_{L^\infty(\Omega)}.\end{aligned}$$

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L^∞-error estimate for OCP

Theorem

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$$\|\bar{p}_h - \bar{p}\|_{L^\infty(\Omega)} \le ch^2|\log h| \left(\|\bar{u}\|_{C^{0,\sigma}(\bar{\Omega})} + \|y_d\|_{C^{0,\sigma}(\bar{\Omega})}\right)$$

are valid.

Proof:

$$\|\bar{y} - \bar{y}_h\|_{L^\infty(\Omega)} = \|S\bar{u} - S_h\bar{u}_h\|_{L^\infty(\Omega)}$$
$$\le \|S\bar{u} - S_h\bar{u}\|_{L^\infty(\Omega)} + \|S_h\bar{u} - S_hR_h\bar{u}\|_{L^\infty(\Omega)} + \|S_hR_h\bar{u} - S_h\bar{u}_h\|_{L^\infty(\Omega)}.$$

FE - error

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L^∞-error estimate for OCP

Theorem

On a family of meshes with grading parameter $\mu < \frac{\lambda}{2}$ the estimates

$$\|\bar{y}_h - \bar{y}\|_{L^\infty(\Omega)} \leq ch^2 |\log h| \left(\|\bar{u}\|_{C^{0,\sigma}(\bar{\Omega})} + \|y_d\|_{C^{0,\sigma}(\bar{\Omega})} \right)$$

$$\|\bar{p}_h - \bar{p}\|_{L^\infty(\Omega)} \leq ch^2 |\log h| \left(\|\bar{u}\|_{C^{0,\sigma}(\bar{\Omega})} + \|y_d\|_{C^{0,\sigma}(\bar{\Omega})} \right)$$

are valid.

Proof:

$$\|\bar{y} - \bar{y}_h\|_{L^\infty(\Omega)} = \|S\bar{u} - S_h\bar{u}_h\|_{L^\infty(\Omega)}$$
$$\leq \|S\bar{u} - S_h\bar{u}\|_{L^\infty(\Omega)} + \|S_h\bar{u} - S_hR_h\bar{u}\|_{L^\infty(\Omega)} + \|S_hR_h\bar{u} - S_h\bar{u}_h\|_{L^\infty(\Omega)}.$$

Supercloseness

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Supercloseness

The approximate solution $\bar{u}_h$ is closer to the interpolant $R_h\bar{u}$ than to the solution $\bar{u}$:

[Apel, Rösch, Winkler, 2005]

On a family of meshes with grading parameter $\mu < \lambda$ the estimate

$$\|\bar{u}_h - R_h\bar{u}\|_{L^2(\Omega)} \leq ch^2 \left(\|\bar{u}\|_{L^\infty(\Omega)} + \|y_d\|_{L^\infty(\Omega)}\right)$$

holds true.

Further, one has

$$\|S_h\|_{L^2(\Omega)\to L^\infty(\Omega)} \leq C$$

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L^∞-error estimate for OCP

Theorem

On a family of meshes with grading parameter $\mu < \frac{\lambda}{2}$ the estimates

$$\|\bar{y}_h - \bar{y}\|_{L^\infty(\Omega)} \le ch^2 |\log h| \left(\|\bar{u}\|_{C^{0,\sigma}(\bar{\Omega})} + \|y_d\|_{C^{0,\sigma}(\bar{\Omega})} \right)$$
$$\|\bar{p}_h - \bar{p}\|_{L^\infty(\Omega)} \le ch^2 |\log h| \left(\|\bar{u}\|_{C^{0,\sigma}(\bar{\Omega})} + \|y_d\|_{C^{0,\sigma}(\bar{\Omega})} \right)$$

are valid.

Proof:

$$\begin{aligned} \|\bar{y} - \bar{y}_h\|_{L^\infty(\Omega)} &= \|S\bar{u} - S_h\bar{u}_h\|_{L^\infty(\Omega)} \\ &\le \|S\bar{u} - S_h\bar{u}\|_{L^\infty(\Omega)} + \|S_h\bar{u} - S_hR_h\bar{u}\|_{L^\infty(\Omega)} + \|S_hR_h\bar{u} - S_h\bar{u}_h\|_{L^\infty(\Omega)}. \end{aligned}$$

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Regularized Dirac function

Let us define a regularized Dirac function for a fixed $a \in T$ as a function with the properties

1. $(\delta^h(a), v_h) = v_h(a) \qquad \forall v_h \in V_h,$
2. $\mathrm{supp}\ \delta^h(a) \subset \bar{T},$
3. $\delta^h(a) \in \mathcal{P}_1(T),$
4. $\|\delta^h(a)\|_{L^2(T^*)} = O(h_T^{-1}).$
5. $\|\delta^h(a)\|_{L^\infty(T^*)} \leq C|T|^{-1}$

An example of such a function is [Scott, 73]

$$\delta^h(x) = \begin{cases} |T|^{-1}\hat{\varphi}\left(F^{-1}(x)\right) & \text{if } x \in T^* \\ 0 & \text{if } x \notin T \end{cases}$$

with $F : \hat{T} \to T$ and $\hat{\varphi}$ a polynomial of degree one, so that

$$\int_{\hat{T}} \hat{p}\hat{\varphi}\ \mathrm{d}\hat{x} = \hat{p}\left(\hat{a}\right) \qquad \forall \hat{p} \in \mathcal{P}_1(\hat{T}).$$

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Regularized Green function

The regularized Green function z^h is defined as solution of

$$a(v, z^h) = (\delta^h(a), v) \qquad \forall v \in V.$$

We denote by z_h^h its discrete counterpart,

$$a(v_h, z_h^h) = (\delta^h(a), v_h) \qquad \forall v_h \in V_h.$$

Here a ist the bilinear form $a(u, v) = \int_\Omega \nabla u \nabla v$.

Lemma

The estimate

$$\|z_h^h\|_{L^\infty(\Omega)} + \|z_h^h\|_{H_0^1(\Omega)} \leq C|\log h|$$

holds on a finite element mesh with $\mu < \lambda$.

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Estimate for $\|S_h\bar{u} - S_h R_h \bar{u}\|_{L^\infty(\Omega)}$

Lemma

The inequality

$$\|S_h\bar{u} - S_h R_h \bar{u}\|_{L^\infty(\Omega)} \le Ch^2 |\log h| \left(\|\bar{u}\|_{L^\infty(\Omega)} + \|\bar{y}_d\|_{L^\infty(\Omega)}\right)$$

is satisfied.

Proof: For an arbitrary, but fixed $a \in \Omega$, we find

$$\begin{aligned} |S_h\bar{u}(a) - S_h R_h \bar{u}(a)| &= |(\delta^h(a), S_h\bar{u} - S_h R_h \bar{u})| \quad \text{(Definiton of } \delta_h(a)\text{)} \\ &= |a(S_h\bar{u} - S_h R_h \bar{u}, z_h^h)| \quad \text{(Definiton of } z_h^h\text{)} \\ &= |(z_h^h, \bar{u} - R_h\bar{u})| \quad \text{(Test function } z_h^h\text{)} \end{aligned}$$

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Estimate for $\|S_h\bar{u} - S_hR_h\bar{u}\|_{L^\infty(\Omega)}$

Lemma

The inequality

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Proof: For an arbitrary, but fixed $a \in \Omega$, we find

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Auxiliary result [Apel, Rösch, Winkler, 2005]:
On a mesh with grading parameter $\mu < \lambda$ the estimate

$$(v_h, \bar{u} - R_h\bar{u})_{L^2(\Omega)} \leq Ch^2 \left(\|v_h\|_{L^\infty(\Omega)} + \|v_h\|_{H_0^1(\Omega)}\right) \left(\|\bar{u}\|_{L^\infty(\Omega)} + \|\bar{y}_d\|_{L^\infty(\Omega)}\right)$$

can be proved for all $v_h \in V_h$.

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Estimate for $\|S_h\bar{u} - S_hR_h\bar{u}\|_{L^\infty(\Omega)}$

Lemma

The inequality

$$\|S_h\bar{u} - S_hR_h\bar{u}\|_{L^\infty(\Omega)} \le Ch^2|\log h| \left(\|\bar{u}\|_{L^\infty(\Omega)} + \|\bar{y}_d\|_{L^\infty(\Omega)}\right)$$

is satisfied.

Proof: For an arbitrary, but fixed $a \in \Omega$, we find

$$\begin{aligned} |S_h\bar{u}(a) - S_hR_h\bar{u}(a)| &= |(\delta^h(a), S_h\bar{u} - S_hR_h\bar{u})| \quad \text{(Definiton of } \delta_h(a)\text{)} \\ &= |a(S_h\bar{u} - S_hR_h\bar{u}, z_h^h)| \quad \text{(Definiton of } z_h^h\text{)} \\ &= |(z_h^h, \bar{u} - R_h\bar{u})| \quad \text{(Test function } z_h^h\text{)} \end{aligned}$$

Auxiliary result + estimate for regularized Green function $\Rightarrow$

$$\begin{aligned} |S_h\bar{u}(a) - S_hR_h\bar{u}(a)| &\le Ch^2 \left(\|z_h^h\|_{L^\infty(\Omega)} + \|z_h^h\|_{H_0^1(\Omega)}\right) \left(\|\bar{u}\|_{L^\infty(\Omega)} + \|\bar{y}_d\|_{L^\infty(\Omega)}\right) \\ &\le Ch^2|\ln h| \left(\|\bar{u}\|_{L^\infty(\Omega)} + \|\bar{y}_d\|_{L^\infty(\Omega)}\right) \end{aligned}$$

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L^∞-error estimate for OCP

Theorem

On a family of meshes with grading parameter $\mu < \frac{\lambda}{2}$ the estimates

$$\|\bar{y}_h - \bar{y}\|_{L^\infty(\Omega)} \leq ch^2 |\log h| \left(\|\bar{u}\|_{C^{0,\sigma}(\bar{\Omega})} + \|y_d\|_{C^{0,\sigma}(\bar{\Omega})} \right)$$

$$\|\bar{p}_h - \bar{p}\|_{L^\infty(\Omega)} \leq ch^2 |\log h| \left(\|\bar{u}\|_{C^{0,\sigma}(\bar{\Omega})} + \|y_d\|_{C^{0,\sigma}(\bar{\Omega})} \right)$$

is valid.

Proof:

$$\begin{aligned}
\|\bar{y} - \bar{y}_h\|_{L^\infty(\Omega)} &= \|S\bar{u} - S_h\bar{u}_h\|_{L^\infty(\Omega)} \\
&\leq \|S\bar{u} - S_h\bar{u}\|_{L^\infty(\Omega)} + \|S_h\bar{u} - S_hR_h\bar{u}\|_{L^\infty(\Omega)} + \|S_hR_h\bar{u} - S_h\bar{u}_h\|_{L^\infty(\Omega)} \\
&\leq ch^2 |\log h| \left(\|\bar{u}\|_{C^{0,\sigma}(\bar{\Omega})} + \|y_d\|_{C^{0,\sigma}(\bar{\Omega})} \right)
\end{aligned}$$

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Postprocessing for the control

Postprocessing step [Meyer, Rösch, 2004]

$$\tilde{u}_h = \Pi_{[a,b]}\left(-\frac{1}{\nu}p_h\right).$$

Theorem

On a family of meshes with grading parameter $\mu < \frac{\lambda}{2}$ the inequality

$$\|\bar{u} - \tilde{u}_h\|_{L^\infty(\Omega)} \leq Ch^2|\ln h|\left(\|\bar{u}\|_{C^{0,\sigma}(\bar{\Omega})} + \|y_d\|_{C^{0,\sigma}(\bar{\Omega})}\right)$$

is satisfied.

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Numerical example (by G. Winkler)

Consider

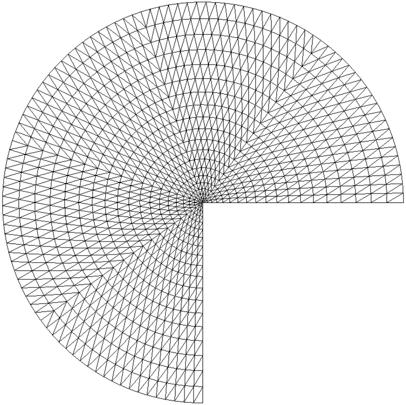
$$
\begin{aligned}
-\Delta y + y &= u + f \quad \text{in } \Omega \\
-\Delta p + p &= y - y_d \quad \text{in } \Omega \\
u &= \Pi_{[-0.3,1]}\left(-\frac{1}{\nu} p\right)
\end{aligned}
$$

with $\nu = 10^{-4}$ and homogeneous Dirichlet boundary conditions for y and p. The data f and y_d are chosen such that

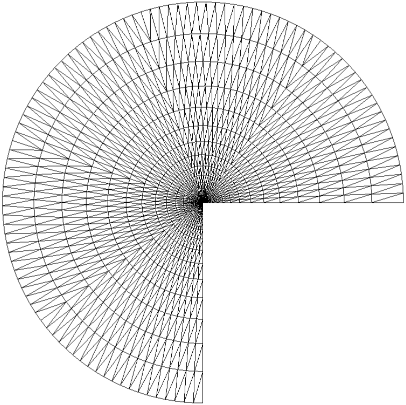
$$
\begin{aligned}
y(r,\varphi) &= \left(r^{2/3} - r^{5/2}\right) \sin \frac{2}{3}\varphi \\
p(r,\varphi) &= \nu \left(r^{2/3} - r^{5/2}\right) \sin \frac{2}{3}\varphi
\end{aligned}
$$

are the exact solutions of the optimal control problem.

Different mesh grading

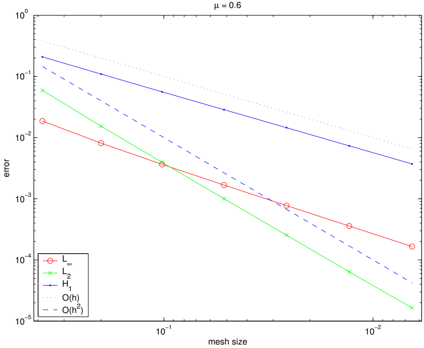


$\mu = 0.6$

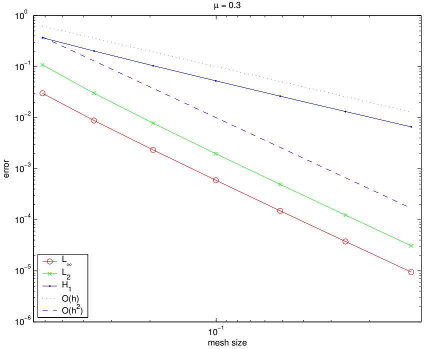


$\mu = 0.3$

Errors in the state for $\mu = 0.6$ and $\mu = 0.3$



$\mu < \lambda$



$\mu < \frac{\lambda}{2}$

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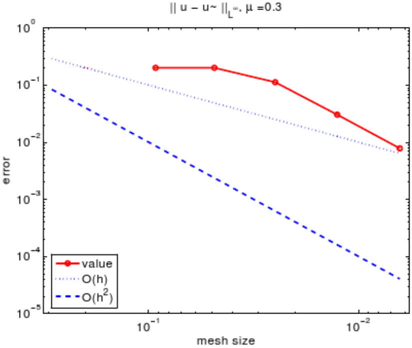
Error in the control for $\mu = 0.3$



ndof	$\|u - \tilde{u}_h\|_{L^\infty(\Omega)}$	
425	2.00e-01	......
1617	1.12e-01	0.84
6305	3.02e-02	1.89
24897	7.77e-03	1.96

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Conclusion and Outlook

Conclusion

- L^∞-error estimate for Dirichlet problem in non-convex domains
- L^∞-error estimate for linear-quadratic OCP in non-convex domains

Current work

Extension of the results to 3D

- L^2-error estimates in polyhedral domains with reentrant edge
  $\rightarrow$ GAMM
- open: L^∞-error estimate for three-dimensional, non-convex domains.