Data envelope fitting with constrained polynomial splines

Abstract:

Estimation of support frontiers and boundaries often involves monotone and/or concave edge data smoothing. This estimation problem arises in various unrelated contexts, such as optimal cost and production assessments in econometrics and master curve prediction in the reliability programs of nuclear reactors. Very few constrained estimators of the support boundary of a bivariate distribution have been introduced in the literature. They are based on simple envelopment techniques which often suffer from lack of precision and smoothness. Combining the edge estimation idea of Hall, Park and Stern with the quadratic spline smoothing method of He and Shi, we develop a novel constrained fit of the boundary curve which benefits from the smoothness of spline approximation and the computational efficiency of linear programs. Using cubic splines is also feasible and more attractive under multiple shape constraints; computing the optimal spline smoother is then formulated into a second-order cone programming problem. Both constrained quadratic and cubic spline frontiers have a similar level of computational complexity to the unconstrained fits and inherit their asymptotic properties. The utility of this method is illustrated through applications to some real datasets and simulation evidence is also presented to show its superiority over the best known methods.