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WEDNESDAY

Jean Paul Van Bendegem

Mathematical practices: walking through the map of the territory

In this first talk the object is to trace the history of the study and philosophy of mathematical practice(s) in order to understand the inevitable heterogeneity of the domain and why this is a desirable feature.

Paul Hasselkuß

What Practices Can (Not) Tell Us About Mathematical Knowledge

What can mathematical practices tell us about mathematical knowledge? By the easy argument, originally due to Lakatos (1976) and Putnam (1979), very much: if mathematicians regularly succeed in establishing some mathematical propositions using quasi-empirical methods, then mathematical knowledge is a posteriori. In recent years, scholars have identified several of such practices. Thus, by the easy argument, mathematical knowledge is a posteriori. But is it? In the talk, I'll argue that the inference from mathematical practice to a posteriority cannot be sustained.

In the first part, I'll briefly discuss two case studies that identify quasi-empirical methods: Tymoczko's (1979) study of computer-assisted proofs and De Toffoli's & Giardino's (2014) study of proofs in knot theory. In the second part, I'll clarify the notions of a posteriority and a priority. Given an agent a who knows p: if p is a priori, then a can also know p a posteriori (e.g. she may have testimonial evidence for p). But, if p is a posteriori, a cannot possibly know p a priori (e.g. p may only be investigated empirically). In the final part, I apply these clarifications to the easy argument. Although Tymoczko et al. do identify practices in which mathematicians know some mathematical facts a posteriori, this does not imply that these facts could not also be known a priori. If correct, this point can be generalized: mathematical knowledge may or may not be a posteriori, but practices alone cannot give an answer in either direction.

Sources:

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Putnam, Hilary. 1979. "What is Mathematical Truth." In Mathematics, Matter and Method, 60–78. Cambridge, Cambridge University Press.

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Frank Scheppers

Monism and epistemic bad faith in Philosophy of Mathematics. Towards a (neo-Wittgensteinian) critical agenda for PMP

This contribution is based on an in-depth study of the critical strands in Wittgenstein's work on mathematics. For the purposes of this talk, I highlight two aspects: (1) content-wise, Wittgenstein's work often opposes the monism that is prevalent in Philosophy of Mathematics (then and now), i.e. the belief that mathematics is a unitary and unique logically consistent system, and (2) Wittgenstein's criticism is typically articulated in ethical/aesthetical terms, specifically in terms of - what I call - epistemic bad faith, i.e. what occurs when someone apparently conforms to (or pretends to conform to) formal criteria for acceptable discourse but at the same time does not participate in those aspects of the encompassing practice that would make such discourse meaningful.

I argue that despite the fact that, in the last few decades, PMP and neighboring fields have accumulated a lot of material that could easily lead to a similarly critical approach to the one demonstrated in Wittgenstein's work, PMP remains remarkably bashful about a number of potentially important philosophical issues with mainstream mathematical discourse, including the question as to how monism about math originated and why it remains prevalent despite there being very little intra-mathematical reason for it.

Fatih Taş

The Philosophy of Mathematics Educators' Mathematical Practices

Many Mathematics educators believe that mathematicians should be concerned with the processes of transferring their research and practice to students. Therefore, educators consider empirical research into the learning and teaching of professional mathematics as a subject for mathematics education research. There is an important commonality between how mathematics educators and philosophers of mathematical practice explore mathematical application. It uses empirical data to make descriptive claims as a basis for making inferences in both groups. However, the types of inferences of each group are different. The implications of mathematics educators ultimately aim to be practical and design instruction and analyze student performance. These aims are naturally related to philosophical issues, as philosophers infer, and often to the epistemological and ontological status of mathematical claims/objects.

I argue that mathematics education research on the mathematical applications of mathematics educators can contribute to PMP. The purpose of this talk will be to discuss the following contributions with examples: mathematics education research can confirm or deny philosophers' claims about mathematical applications. Second, mathematics educators' claims about mathematical practice may constrain philosophical explanations of mathematical practice. Third, the surprising results from mathematics education studies on mathematical application could give philosophers interesting topics to explain.

Mikkel Tvorup Moseholm

An Agile Approach to the Philosophy of Mathematical Practice

In this talk we discuss challenges of interdisciplinary work in the Philosophy of Mathematical Practices (PMP) and argues that the current institutional setting of philosophy (both analytic or continental) brings epistemic problems to this endeavor. For example: The field is closely linked to many disciplines, like philosophy, history, sociology, and mathematics education. This makes it difficult for scientists engaging in PMP to navigate different value and credit systems across disciplines, i.e. to get their career needs and the scientific needs aligned.

We argue that this situation of interdisciplinary work and scope of the object we study invites institutional epistemological-interference. If the subject matter cannot be studied by individual 'arm chair philosophers', then it would require interdisciplinary collaboration and the creation of more encompassing views as strictly necessary for studying mathematical practice. To achieve this, the talk proposes adopting methods and organizational structures known from software development.

More concretely, we argue that the framework of classical philosophy resembles what is called the \textit{waterfall model} in software development. This model is characterized by breaking down the software development process into sequential steps. Advancement to the next step is dependent on fulfilling the requirements in the previous step.

We contrast this with the basic idea of iterative and incremental software development (IID), where the process is to develop and build the software through repeated iterations and in well-defined increments. Most importantly, this allows the developer to incorporate what she learned in the previous cycle.

This is joint work with (TBA)

Konstantinos Konstantinou

Is Mathematics Safe?

Recently, Justin Clarke-Doane (2020) has argued that the Benacerraf-Field challenge is best cashed out as the challenge to show that mathematical beliefs are safe, i.e., that they could not have easily been false. This modal condition amounts to saying that, in all nearby possible worlds, our mathematical beliefs are true. But how could one embark on showing that this condition is met?

In this talk, I propose that we can make progress in addressing the challenge by focusing on mathematical practice. To do so, our attention should be directed to the beliefs of the mathematical community as a whole, in the spirit of social epistemology, as opposed to individual mathematicians' beliefs. Against this background, we can hope to derive the safety of mathematical beliefs from the justificatory norms of the mathematical community.

One of those norms is the formalizability constraint, according to which every acceptable mathematical proof can be "decompressed" into a deductively valid formal derivation (see Azzouni, 2004). This constraint, I want to suggest, provides the kind of "local necessity" (Williamson, 2009) that the safety condition imposes on mathematical beliefs. More specifically, by respecting the ideal that each step in a correct proof corresponds to a series of steps in a valid formal derivation, mathematicians ensure that their inferences could not have easily been mistaken.

The above project seems a promising case study of how focusing on actual mathematical practices has the potential to illuminate metaphysical and epistemological riddles from the "classical" 20th-century philosophy of mathematics.

Carole Hofstetter

Aristotle as a reader of Euclid and Nicomachus? On a Byzantine mathematical practice of composition

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The medieval Byzantine period constitutes a corpus that has been little explored in the philosophy of mathematics. I would like to focus on a mathematical compendium composed by a Byzantine scholar, Neophytos Prodromenos (14th century). It is composed of extracts from Euclid's Elements and Nicomachus of Gerasa's Introduction to Arithmetic (2nd century CE).

This compendium is placed at the head of the Prior Analytics in the fourteenth-century manuscript that transmits it, which suggests that it is supposed to introduce the study of Aristotle's treatise. Yet the wording of its title is extremely curious and indicates an unexpected understanding of the compositional process of the Aristotelian treatise.

I will first focus on the title. First, I will seek to determine what its formulation implies about the medieval scholar's understanding of the Aristotelian treatise. Independently of any historical considerations, I will then explore the extent to which Prodromenos designates Aristotle's approach as an enterprise of designing a mathematical object and using it in the service of its own purpose.

Furthermore, I will look at the mathematical practice that is exemplified by this medieval composition. Prodromenos' choice to collect these texts and to relate them to philosophical passages suggests a reading of philosophy that involves a practice of mathematics. What conception and understanding of mathematical language does this reflect? The question arises when one observes that Euclid provides a proof by recourse to demonstration, while Nicomachus does so by experimentation with his assertion.

THURSDAY

Jean Paul Van Bendegem

One more time: are there experiments in mathematics?

In this second talk I will address (once more time) the conundrum whether or not it makes sense to speak of mathematical experiments, whether real or imaginary. And why this question is, philosophically speaking, important.

Daniel Rowe

Finitistic Temporal Tools for the Disambiguation of Paradoxes

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There are philosophically important reasons to prefer finitism to the actual-infinitistic accounts of classical mathematics. But in this talk I wish to argue for a technical benefit too. I claim that finitistic approaches lend themselves to temporal logic. I then argue that two-dimensional temporal logic can be used to disambiguate self-referential paradoxes.

I briefly show the origins of this strategy in recent specialised work on contrary-to-time conditionals in ancient legal contexts (Abraham, Gabbay & Schild, 2011, 2012 and 2013) and my own work with Dov Gabbay (2023a, 2023b).

I focus on self-referential semantic paradoxes, (liar sentences and multi-person liar paradoxes). My claim (based upon my recent submission to Synthese) is that the use of Two-dimensional Temporal Logic offers an important improvement upon the contextualist approaches of (Parsons (1974), Barwise & Etchemendy, (1987), Simmons (1993), Glanzberg (2001, 2004)) resistant to standard critiques (eg. Gauker 2006).

Finally I would like to sketch how the generalisation of this strategy could permit complete second-order logic in standard semantics. Currently there are completeness theorems for second-order logic in Henkin semantics (Henkin 1950). The latter has much similarity to type theory and is in a sense akin to a multi-layered first-order language. The possibility that a two-dimensional temporal logic could be complete using standard semantics, would offer various forms of potentialism and finitism not merely philosophical precisification of actualistic mathematics, but the possibility of gaining the full expressive power of complete second order logic in a well-motivated way.

Takahiro Yamada

A Classical Reconstruction of Wright's Strict Finitistic First-Order Logic

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Crispin Wright in his 1982 paper argues for strict finitism, a constructive view of mathematics that is more restrictive than intuitionism. According to the intuitionist, a mathematical statement is accepted if it is verifiable in principle; and a number is accepted if it is constructible

in principle. The strict finitist's tenet is that one should replace `in principle' by `in practice'. Then the totality of the numbers shall be finite, since one could not actually construct infinitely many numbers.
In his strict finitistic metatheory, Wright sketches models of strict finitistic arithmetic which are tree-like structures, and thereby proposes a model-theoretic explanation of the strict finitistic logical connectives. We will in this talk present our classical interpretation of his semantics, and show formal properties of `strict finitistic logic', the abstract system of strict finitistic reasoning.
Our result shows that while the logic lacks the law of excluded middle and Modus Ponens, it succeeds in formalising some of Wright's conceptual assumptions on strict finitism. One is that $\neg A \vee \neg\neg A$ is valid, which would mean that any statement is either practically verifiable or not. Further, a strict finitistic model can be viewed as a node in an intuitionistic model if some conditions are added. We will provide a result that could be interpreted as a formalisation of the conceptual relation with intuitionism that a statement is verifiable in principle iff it is verifiable in practice with some extension in practical resources.

Ariles Remaki

Synchronic tables and diachronic formulas - study of the mathematical practices of Leibniz

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If one pays close attention to the way Leibniz conceives simple notions such as numbers, relations, time and also space, il appears that space has a special place in these considerations. This is because of a neglected aspect of Leibniz's method that I brought to light during my PhD: tables. These combinatorial structures played an important role in Leibniz's mathematical work, and this role has been overshadowed by the question of symbolism and the formal reading of Leibniz's mathematical writings. Yet, the philosophical conception of space and time that Leibniz developed in his maturity has very interesting analogies with the considerations that one has to put in place when examining this methodological distinction between tables and formulas. Like time, formulas are sequences of characters that make sense through a linear, anisotropic and univocal structure. Whereas tables use a two-dimensional space to express meaningful relationships: the structure is isotropic and ambiguous. Indeed, tables can be read in several different ways, and this equivocality was useful to the young Leibniz and his ars inveniendi.
The analogy between the two couples space/time and table/form is not historically relevant, as recent studies on Leibniz's work have shown that his conception of space and time had followed quite different paths from those of the ars inveniendi. Nevertheless, this gives us interesting philosophical tools to better understand the mathematical practices of modern scholars.

Nicolas Nam Millot

The structure of perceptual space

In this talk, I suggest an outline of the genesis of our geometrical intuitions starting from the visual and sensory-motor experience. The main idea is to link two distinct research programs: on the one hand the research of the cognitive foundations of the phenomenological experience of space, and on the other hand the investigation of cognitive history in the sense of Reviel Netz. In our opinion, it is possible to retrace this path in 4 stages:
1) A description of the structure of our perceptual space at its different levels of structuring, starting from a description of the structure of our sensorimotor and visual system (inspired by the work of Koenderink and Petitot).
2) A description of the link between the phenomenal structure of our sensorimotor and visual space and the syntactic structure of our language.
3) A description of the phenomenology (in the sense of discourse about phenomena) of visual objects (figures and images), which allows us to identify the structure of shapes of visual objects.
4) A description of how this phenomenology is the cognitive condition of our manipulation of geometric figures and mathematical diagrams, and how it specifies geometric intuition as a kind of act distinct from the "raw" perception of figures.
We will concentrate mainly on the last two points, which seem to be the most interesting for the philosophy of mathematical practice since they allow us to specify, more intimately than is usually done, the type of genesis that conditions the construction of our geometric knowledge.

Tristan Tondino
Perspective and Artistic Practice

In recent years, Aesthetics has branched into many fascinating sub-fields. Philosophy of Film is an excellent example. This paper grows out of Nelson Goodman's analysis of perspective in Languages of Art, and our interest here is what may be construed as a Philosophy of Artistic Practice with goals like those found in other burgeoning fields i.e., the Philosophy of Mathematical Practice[1]. Beginning from Nelson Goodman's conventionalist approach to perspectival representation[2], we analyze several recent contributions, and offer as an alternative, a weakened conventionalism.

Jean-Charles Pelland
Mathematical practices as cognitive chaîne opératoires

While it sounds almost tautological to say that the notion of 'practice' is at the heart of the philosophy of mathematical practice, what exactly practices are supposed to be is far from obvious (Turner 1994; Rouse 2006). To make matters worse, there are many ways to apply this notion to the philosophy of mathematics more broadly (e.g. Carter 2019; Van Bendegem 2014). One ambiguity lies in whether or not the notion of 'practice' is best thought of as applying to individuals or to cultural groups (Reckwitz 2002; Rouse 2003). In this talk I argue against the latter option by highlighting problems the cultural interpretation faces in explaining the origins, development , and diversity of numeration practices. As a solution to this problem, I explore the advantages of borrowing the notion of 'chaîne opératoire' from archeology. Importantly, this means that the notion of practice, when applied to mathematics, involves sequences of both material and cognitive actions that, in some cases, may be implemented by more than one individual. One upshot of seeing practices as sequences of cognitive actions undertaken towards a specific goal will be that we can track both individual contribution to the historical development of mathematical methods and proofs as well as the cognitive and material tools required for these. This perspective also allows us to highlight common elements of similar practices and identify where they diverge.

FRIDAY

Jean Paul Van Bendegem
Contingency in mathematics: the relevance of 'What if' stories.

In this third talk the object is not to address the full question whether mathematical knowledge is necessary or contingent. That theme is too vast, but a more interesting theme is whether it is makes sense to talk about alternative developments in the history of mathematics.

Franci Mangraviti
Critical maths kinds

Mathematics, even more than the other sciences, is often presented as essentially unique, as if it could not be any other way. And yet, prima facie alternative mathematics seem to exist, or at least be possible: even without leaving the boundaries of Western mathematics, we have intuitionistic mathematics, alternative set theories, all kinds of inconsistent mathematics, and more. The existence and status of alternative mathematics has already been shown to be philosophically significant, most notably due to connections with social constructivism, feminist epistemology, the contigentist/inevitabilist debate, and logical pluralism. Despite this, to my knowledge no general ways to approach the topic of alternativeness have been proposed, with different authors seemingly using "alternative" in quite different ways depending on their particular goals. In this talk I will introduce a practice-centered framework for discussing alternative mathematics - inspired by Robin Dembroff's notion of critical gender kind, and by Andrew Aberdein and Stephen Read's discussion of alternative logics - understanding alternatives as variations from a standard model along several possible dimensions and with several possible revolutionary intents. After sketching a model of "dominant" mathematics, I will discuss some examples of how the framework can be of help in thinking through questions concerning the alternativeness status and philosophical implications of alleged alternative mathematics.

José Antonio Pérez-Escobar

Building on the late Wittgenstein to account for the applicability of mathematics

The applicability of mathematics in science has received renewed attention in recent years. The two most influential recent approaches are the mapping account (Pincock, 2004) and the inferential account (Bueno and Colyvan, 2011). The mapping account establishes structural parallelisms between empirical phenomena and mathematics at the base of applicability, while the inferential account builds on the former by adding pragmatic inferences in a three-step model (immersion-derivation-interpretation). The latter pragmatic inferences account for cases like the following: simple functions of displacement of projectiles in physics are quadratics with two solutions (a positive and a negative one), but only one solution is physically meaningful. I argue that the inferential approach leaves further inferential, representational and normative aspects of the applicability of mathematics in practice unaddressed. To account for such gaps, I build on three features of late Wittgensteinian philosophy, namely inferential restrictiveness at the formal derivation level, mathematics as rules of description, and hinge epistemology, and incorporate them to the standard inferential approach. By doing this we further understand the pragmatics of the applicability of mathematics in science. I will place especial emphasis on 1) how mathematics can normatively guide measurement in science (thereby adjusting the fit between empirical observations and mathematical structures) analyzing two scientific cases, and 2) how there is leeway in the formal derivations performed on mathematical models in practice (some that some results are not just discarded as "meaningless", but do not appear in the first place) analyzing overlooked aspects of Bueno and Colyvan's hallmark example: Dirac's equation.

Baptiste Guarry-Petit

Group theory and bricolage : mathematical practices beyond the borders of literacy

Claude Lévi-Strauss's Elementary structures of kinship (Lévi-Strauss 1949) are known as the first use of mathematical models to give an anthropological account of traditionnal kinship systems, a method applying group theory that then spread in ethnomathematics and mathematical anthropology (Rauff 2016). But a question remained : what connection could be drawn between these analytic possibilities and the way the systems were elaborated in the first place ? As Lévi-Strauss's research became famous as aiming to discover the laws of thought, such systems could be understood as pure but unconscious products of the human mind. Such is the "mentalist" anthropology that Jack Goody's criticism reached, asking us to come back to practices, and especially to the "technologies of the intellect" (Goody 1977). Lévi-Strauss did not respond to Goody, but he already did to Jean-Paul Sartre's similar criticism (Sartre 1960) in a way that made him rethink the origin of kinship systems and their mathematical properties. The savage mind (Lévi-Strauss 1962) thus presents the origin of such systems as a practical one, comparing it to some intellectual bricolage. In this talk, I would like to show how mathematical practice can then be understood beyond the borders of literacy : the mathematical accounts of sign systems can indeed be linked to their practical production. The bricolage, conceived as the practice of putting together a finite set of given artifacts in diverse ways, is a combinatorial practice, and we can easily use group theory to interprete its different results as a transformation group.

Deborah Kant

Deep disagreement in set theory

Set-theoretic practice provides us with a philosophically interesting case of both a deep and peer disagreement. The standard theory of sets, ZFC, is incomplete in a mathematically relevant sense: many set-theoretic questions are left unanswered. Facing this situation, set-theoretic practitioners hold quite different, but often determinate, views on this issue. Some adopt an absolutist view; they believe that further axioms can be justified such that some of the independent sentences are proven or disproven. Others have a pluralist view; they believe that this is not possible, that no further axioms besides the ZFC-axioms can be justified and that one has to accept the independence phenomenon as inherent to set theory. The description of the case study is based on detailed empirical data gathered in an interview study with set-theoretic practitioners.

It is a case of a deep disagreement, because the parties disagree about epistemic principles: reasons in favour of a new axiom candidate that are accepted by set theorists with an absolutist view are not accepted as reasons by set theorists with a pluralist view. Moreover, many set-theoretic practitioners are epistemic peers. Hence, the conciliationist challenge is raised: should the parties to the disagreement change their beliefs? Based on my analysis of this case, I argue that an answer to the conciliationist challenge depends on the considered epistemic goal: either the parties of the disagreement aim at mathematical progress and should remain steadfast or they aim at true beliefs about the propositions on which they disagree and should suspend judgement.