Program – Masterclass in the Philosophy of Mathematical Practices with Catarina Dutilh Novaes

Monday, June 24: Room D.2.06

13.00–13.15: Welcome

13.15–15.00: LECTURE 1, **The Dialogical Roots of Deduction**

15.00–15.30: Coffee break

15.30–16.00: Jordi Fairhurst Chilton, **Why Deep Disagreements Are of Interest to Mathematics**

16.00–16.30: Lorenzo Gandolfi, **The Epistemology of Reductio Ad Absurdum**

16.30–17.00: Coffee break

17.00–17.30: Mourtaza Chopra, **Architecture in Early Greek Geometry**

17.30–18.00: Mary Elworth, **Visual representations and conceptualizations: cluster algebras as a case study**

18.00–18.15: Short break

18.15–18.45: Emelia Stanley, **Mathematical Analogy and Open Texture**

(Chairs: Colin, Guido, and Line)

Tuesday, June 25: Room D.2.06

09.00–12.00: LECTURE 2, **Dialogical Pragmatism**

(including one or two short coffee breaks)

12.00–12.30: Vasiliki Xiromeriti, **Formal Theories of Collective Deliberation: Do Bayesians Argue?**

12.30–13.30: Lunch

13.30–14.00: Fatih Taş, **Mathematical Explanations and Understanding in Mathematics Education**

14.00–14.30: Guido Löhr, **Could They Have Turned Out Differently?**

14.30–15.00: José Antonio Pérez-Escobar, **Turing, Wittgenstein and Structural Analogies in the Lectures on the Foundations of Mathematics**

15.00–15.30: Coffee break

15.30–16.00: Thomas Glasman, **Two Kinds of Mathematical Naturalism**

16.00–16.30: Andrej Jovićević, **Fictionalism and Counterpossibles**

(Chairs: Deniz, Line and Lorenzo)

Wednesday, June 26: Room D.2.06

09.00–12.00: LECTURE 3, **Definitions and Conceptual Creativity in Mathematics**

(including one or two short coffee breaks)

12.00–12.30: Mireia Martínez i Sellarès, **When are two geometrical figures affine? On a disagreement between Euler and Möbius**

12.30–13.00: Paul Hasselkuß, **What's The Problem With Computer-Assisted Proofs?**

(Chairs: Colin and Deniz)

LECTURE 1, **The Dialogical Roots of Deduction**

This lecture introduces the main ideas developed in my monograph The Dialogical Roots of Deduction (CUP, 2020) henceforth DRD. The main idea is that a deductive argument such as a mathematical proof can be understood as a dialogue between two (fictive) participants, Prover and Skeptic. Prover aims to produce explanatory persuasion, while Skeptic aims to ensure that the proof is valid and perspicuous. Thus seen, the ideal of explanatoriness for mathematical proofs is naturally understood as a dialogical desideratum. In the book, I show that the dialogical perspective allows for a unified account of deductive theories and practices, explaining with one stroke various puzzling and apparently disconnected features of the relevant phenomena.

Jordi Fairhurst Chilton (Universitat de les Illes Balears), **Why Deep Disagreements Are of Interest to Mathematics**

Deep disagreements are systematic and persistent disagreements rooted in conflicting fundamental beliefs, epistemic principles and/or values, where there is no further epistemic principle or evidence accepted by all parties to settle the dispute. Recently philosophers have shown interest in mathematical deep disagreements (Aberdein 2023; Kant 2023; Wagner 2023). These disputes are instances in which mathematicians, despite sharing epistemic goals (e.g., bringing true beliefs in mathematics), are faced with persistent disagreements with no clear path to resolution because they use different epistemic principles to attain these goals. These peculiar disagreements may pose a threat to mathematical progress. They are instances in which mathematicians' epistemic principles take them in different directions, thus preventing them from going together in the attainment of their shared epistemic goals. Building on De Cruz and De Smedt's (2013) research on the epistemic value of deep disagreements in natural sciences, this paper explores why mathematical deep disagreements may be of interest for progressing mathematics and mathematical practice. Specifically, it argues that by adopting an attitude of intellectual humility, mathematicians can use these disputes to generate new evidence (e.g., proofs), better understand the assumptions underlining their mathematical practices and combat confirmation bias, thereby forwarding the mathematical enterprise.

Lorenzo Gandolfi (University of Turin), **The Epistemology of Reductio Ad Absurdum**

When a mathematician decides to use reductio ad absurdum to prove that a statement p is a theorem, she starts negating p, wanting to show that not-p, together with some additional assumptions, leads to an unacceptable conclusion and that p is therefore true. It seems plausible to suppose that when she chooses to employ such procedure she has some kind of preliminary evidence, whatever it actually looks like, making her degree of belief for p relatively high. Nevertheless, she acts against her evidence assuming that not-p. It is quite difficult to account for the rationality of this behavior by means of epistemology. On the one hand, we observe an apparent conflict arising between an epistemic norm, telling the mathematician that she should form a belief that p, and a zetetic (i.e. inquiry-related) norm, telling her she should assume that not-p. On the other, it is not obvious what kind of attitude an assumption is, how (if at all) it differs from a belief, and which criterion we should use to assess its rationality. After examining several hypotheses on what the mathematician's preliminary evidence could consist in, accepting Friedman's (2020) suggestion that zetetic norms could be epistemic norms, I argue that assumptions are to be considered a specific subset of transitional attitudes in the sense of Staffel (2021). However, I maintain that we should revise

Staffel's proposed criterion for assessing the rationality of transitional attitudes (i.e. pro tem rationality), since it is not suitable for assumptions made against the evidence.

Mourtaza Chopra (Tel Aviv University), **Architecture in Early Greek Geometry**

Surprisingly, the importance of architecture in the emergence of early Greek geometry is well-known, for instance for the requirement of doubling the altar in Delos, but understudied. I want to present a work in progress that I have just begun in my present post-doctoral fellowship. My main goal is to understand what early Greek geometry owes to architectural practices and to submit some hypotheses on Greek geometrical constructions. This involves three questions asked in the context of early Greek geometry:

- In what sense are drawing and designing properly constructive activities? I suggest that geometric drawing follows the laws of planning for a possible construction with masonry tools.

- In what way Greek geometrical constructions are specific in regard to other kinds of early constructions? The term diagram should specified. It seems to me that the striking feature is the importance of contacts and intersections which give rise to the theoretical concepts of angles and points.

- How can the relationship between geometry and architecture be conceived theoretically, in general and in this special context? I suggest that for temple architecture which appears at the same time, the relevant aspects are those by which the temple appears as a center in the environment and a key to interpret it. These socio-geographical features determines its geometric structure.

All these hypotheses lead me to the conclusions that architectural concerns deeply inform the conceptual framework of early Greek geometry. I strongly hope that those ideas will be discussed by other participants.

Mary Elworth (Aarhus University), **Visual representations and conceptualizations: cluster algebras as a case study**

Mathematical practice makes use not only of ordinary language, but also visual 'representations' or 'signs,' such as diagrams and notation, which play a multifaceted role in practice. For example, signs can be a means to externalize thought, and thus to facilitate the creation of a shared understanding of concepts but also to provide a (physical or material) medium for performing manipulations or operations on mathematical expressions, broadly interpreted. In this talk, I will explore the issue of how we may conceive of various interrelations between concepts, definitions, and forms of visual representations. In particular, I address the question of how distinct forms of representations can influence how a subject matter is conceptualized. This exploration is rooted in a case study involving work on cluster algebras – a recently developed mathematical theory which arose in the early 2000s. Cluster algebras have since taken on a 'life of their own,' that is, with ongoing research endeavors which attempt to systematically investigate numerous 'deep' connections between cluster algebras and a diverse range of other mathematical domains. While cluster algebra theory is barely more than twenty years old, some of the central concepts and corresponding representations deserve close attention for what they may be able to tell us more generally about how concepts and definitions relate to the issue of representations in mathematical practice.

Emelia Stanley (University of Vienna), **Mathematical Analogy and Open Texture**

Reasoning by analogy has always played an essential role in mathematical discovery, education, and proof. Flat-footedly, analogical reasoning involves considering a novel subject, from which conclusions are drawn on the original by applying appropriate translations after inferences have been drawn. In some cases, these analogical steps may be called wholly syntactic; but more commonly, the reasoning proceeds by semantic and even intuitive inferences on the analogical domain.

Open-texture is the notion that certain concepts have cases to which they neither definitely apply nor disapply. While an important term in philosophy of science and jurisprudence, only recently has open-texture been considered in the context of mathematical concepts (Tanswell 2018, Zayton 2022). While seeming to be strongly borne out in the observation of mathematical practice, mathematical open-texture is untuitively unamenble to many philosophical accounts of mathematical truth and knowledge.

I propose that open-texture naturally arises as a consequence of analogical reasoning in mathematics. Considering several case studies, critically the development of the concepts of continuity and function, I show how this reasoning can be responsible for pluralistic mathematical validity and the creation of Lakatosian "monsters." More importantly, I provide an account of how texture is then closed in mathematics, through stochastic social processes, asymmetry of authority between individuals and/or institutions, and the priority of certain mathematical norms. I conclude that analogical reasoning is both threatening to rigour, but critical for mathematical progress.

LECTURE 2, **Dialogical Pragmatism**

There is a sense in which DRD does not address head-on a fundamental philosophical question, namely that of the (transcendental) justification of deduction. What, if anything, can justify deduction? What the book offers instead is an analysis of deduction insofar as it is instantiated and embedded in actual human practices. This approach is motivated by pragmatist commitments, and yet, in DRD, the pragmatist grounding remains only implicit. This lecture rectifies this omission by articulating more explicitly the dialogical pragmatism at the core of my account. Dialogical pragmatism takes a clear stance on the justification question: there is nothing above and beyond the practices of deduction that is relevant for the question of the justification of deduction. To support this claim, I draw a comparison with pragmatist theories of truth, and discuss the recent literature on anti-exceptionalism about logic and what counts as evidence for logical theories.

Vasiliki Xiromeriti (Jean Moulin Lyon 3 University), **Formal Theories of Collective Deliberation: Do Bayesians Argue?**

Despite the growing theoretical and practical interest in collective deliberation during the past decades, little is still known about the mechanisms of collective deliberation, its effects, and the conditions in which these effects are likely to occur. How do agents revise their attitudes (beliefs, preferences, and intentions) in response to the interactions taking place during deliberation? Collective deliberation seems to escape the explanatory power of our best theories of social revision.

The aim of this paper is to examine the ability of formal theories in philosophy, economics, and political science to explain the mechanisms and effects of collective deliberation. I will focus on Bayesian models of social belief revision in epistemology and decision theory. Bayesian models represent how agents rationally revise their beliefs and, on this basis, their (derived) preferences and intentions, after learning information during deliberation. Game theory makes it also possible to represent the conditions under which agents have an interest in withholding information or misrepresenting their positions.

To what extent can Bayesian models account for the complex interactions taking place during collective deliberation? I will begin with a brief review of Bayesian theories of social revision. One may apply these models to account for the informational aspects of collective deliberation. However, collective deliberation is not the kind of reasoning aiming to elicit and aggregate information. One prominent aspect of collective deliberation is the argumentative one. I will propose a taxonomy of the mechanisms of collective deliberation and test how far Bayesian models can go in explaining argumentation.

Fatih Taş (Bartın University), **Mathematical Explanations and Understanding in Mathematics Education**

The connection between mathematical understanding and explanation has recently attracted the attention of philosophers and mathematics teachers. Educators and philosophers largely manage this relationship independently of each other. It is argued that we should regard mathematical explanations as a functional process that generates the content to be taught. In this investigation, I begin with the assumption that this view of mathematical explanation is correct. As a result, I discuss how mathematical explanations are used in the mathematics teaching process and how explaining a mathematical concept affects student understanding. I discuss how mathematical explanations are made in the teaching process and explain their effects on understanding mathematical concepts and the learning process. I argue how important beyond

proof can be when we understand the connection between mathematical explanation and understanding. I expand on their importance in mathematics education by detailing how proofs, definitions, and visualizations can be descriptive. In conclusion, we find that we have a good idea of how attention to the connection between mathematical explanation and understanding makes learning mathematics effective.

Guido Löhr (VU Amsterdam), **Could They Have Turned Out Differently?**

"…the dialogical account of deduction views as futile the search for transcendental grounding for deductive validity in terms of supra-social properties or entities that make (some) arguments deductively valid, i.e., what we might describe as 'validity-makers'."

The key ingredient missing for this account is this: What exactly is the nature of these social practices. How can they generate deductive validity? How do logical rules of inference seem so obvious and impossible to question?

The answer, I argue, must lie in the nature of the normativity of joint action (as opposed to social norms/practices more generally – here is where I potentially disagree with CDN).

I argue that phenomenon of validity is grounded in what in the joint action literature is called "directed duties" (Gilbert, 2018; Roth, 2017). I owe it to you to accept P if I accept P AND Q. What is the nature of these directed duties? My talk will spell this out in more detail (based on Löhr, forthcoming).

A deep dive into the normativity of joint action also gives us insights into the roots of deductive inference rules and why they are the way they are and could not have been otherwise (contrary, I think, to what CDN would argue). They are part of the conditions for cooperation/joint action. Could they have been different? It is possible, but hard to imagine how they could survive cultural evolution/group selection.

Most interesting cases are conditionals (IF P THEN Q, P – THEREFORE Q) and syllogisms. As they make linguistic rules explicit (Brandom, 1994) to challenge them is to challenge social norms and is conceptually highly disruptive (Löhr, 2023).

José Antonio Pérez-Escobar (University of Geneva), **Turing, Wittgenstein and Structural Analogies in the Lectures on the Foundations of Mathematics**

The disagreement between Wittgenstein and Turing during Wittgenstein's lectures in Cambridge has been used to clarify their attitudes towards logic and mathematics (especially Wittgenstein's). The structural analogies employed in their discussion remain underexplored, despite playing a pivotal point in the discussion: they illustrate Turing's arguments for his view of logic and mathematics as structures that in turn mimic empirical structures, and in Wittgenstein's rejection of such a view. In turn, they are key to each one's tolerance (or lack thereof) of contradictions in logic and mathematics, both in pure and applied contexts. Furthermore, both Turing and Wittgenstein attempt to convey and convince each other of their views via these analogies, often to no success, given their limited elaborations on these analogies. I clarify Turing's and Wittgentein's attitudes towards structural intuitions in logic and mathematics by analyzing structural analogies in the Lectures on the Foundations of Mathematics. This leads to a better understanding of their views and clears up some misconceptions about them.

Thomas Glasman (Vrije Universiteit Brussel), **Two Kinds of Mathematical Naturalism**

Mathematical Naturalism describes several different schools of the philosophy of mathematics, but concerning historically inclined accounts, three philosophically inclined accounts dominate. These are Philip Kitcher's, Penelope Maddy's and David Bloor's. While Philip Kitcher's work critically engages with approaches that constitute the 'strong programme', there exists no dialogue between Maddian and Bloorian naturalism.

This presentation will note that, despite there existing essentially no citations between the two leading figures of their schools of naturalism, there are acknowledgements of one another in published work—such as Maddy's condemnation of 'glib sociology'. It will compare these two kinds of naturalism, their assumptions and their goals.

Drawing out the differences between the two, the paper will ask questions as to why such interactions have been limited, and argue that such interaction is necessary. The Maddian approach—in breaking the link between axioms and intrinsic truth—opens up set theory to a Bloorian, or at least 'externalist' critique. By breaking the link between axioms and necessity, Maddy raises the possibility of error, and demands the investigation of why certain axioms and fundamental assumptions were chosen over others.

While Maddy maintains a normative reason for excluding the social from the mathematical, it will be argued that philosophically the breach between the necessary and the mathematical means such investigations become significant. Furthermore, it will be argued that such limitations entail an incomplete historical account.

Andrej Jovićević (KU Leuven), **Fictionalism and Counterpossibles**

Nominalist and platonist positions in philosophy of mathematics admit contrasting accounts of the ontological domain. Since the platonist takes mathematical claims to refer to abstract objects, the nominalist incurs the burden of explaining how mathematical propositions secure their reference in the absence of such entities. Fictionalism is a prominent strategy for meeting this explanatory burden: propositions about abstract mathematical entities are not countenanced categorically, but hypothetically as the antecedents of counterfactual conditionals (e.g. Lewis 1978; Dorr 2005). However, since the standard semantics vacuously verifies all counterfactuals with impossible antecedents (counterpossibles) (Stalnaker 1968; Lewis 1971), all fictionalist counterfactuals are vacuously verified. Recently, the standard semantics has been defended against intuitions suggesting that counterpossibles have non-vacuous truth conditions (see Williamson 2017; 2018; 2020). This result poses significant threats to a fictionalism that relies on non-vacuous readings of counterpossible situations entertained as fictions.

This presentation considers the prospects of fictionalist approaches to platonist positions in philosophy of mathematics in light of the newly sustained case against non-vacuous treatments of counterfactuals. We survey available lines of fictionalist resistance – e.g. endorsing an unorthodox semantics for counterfactuals, modifying the fictionalist strategy – and argue that they are insufficient if they are put forward only as ad hoc repairs. This leads us to a powerful desideratum: any defense of an alternative fictionalism must provide a parallel and connected revision of the standard semantics for counterfactuals. Relying on previous work by Fine (Fine 2017; 2021; 2020) and Yablo (Yablo 2014; 2017; 2016), we suggest that a proposal meeting this desideratum can be formulated via a truth-maker semantics for both (counterfactual) conditionals and fictionalism.

References:

Dorr, Cian. 2005. “What We Disagree about When We Disagree about Ontology.” In Fictionalism In Metaphysics, edited by Mark Eli Kalderon, 234–86. Oxford: Oxford University Press.

Fine, Kit. 2017. “Truthmaker Semantics.” In A Companion to the Philosophy of Language, 556–77. John Wiley & Sons, Ltd. ———. 2020. “Yablo on Subject-Matter.” Philosophical Studies 177 (1): 129–71.

———. 2021. “Constructing the Impossible.” In Conditionals, Paradox, and Probability: Themes from the Philosophy of Dorothy Edgington, edited by Lee Walters and John Hawthorne. Oxford: Oxford University Press.

Lewis, David. 1971. Counterfactuals. Malden: Blackwell Publishers.

———. 1978. “Truth in Fiction.” American Philosophical Quarterly 15 (1): 37–46.

Stalnaker, Robert. 1968. “A Theory of Conditionals.” In Studies in Logical Theory, edited by Nicholas Rescher, 98–112. Blackwell.

Williamson, Timothy. 2017. “Counterpossibles in Semantics and Metaphysics [Special Issue].” Argumenta 2 (2): 195–226.

———. 2018. “Counterpossibles.” Topoi 37 (3): 357–68.

———. 2020. Suppose and Tell: The Semantics and Heuristics of Conditionals. Oxford: Oxford University Press.

Yablo, Stephen. 2014. Aboutness. Carl G. Hempel Lecture Series. Princeton (N.J.) Oxford: Princeton University Press.

———. 2016. “Ifs, Ands, and Buts: An Incremental Truthmaker Semantics for Indicative Conditionals.” Analytic Philosophy 57 (3): 175–213.

———. 2017. “If-Thenism.” Australasian Philosophical Review 1 (2): 115–32.

LECTURE 3, **Definitions and Conceptual Creativity in Mathematics**

Another topic that DRD does not address at length is the issue of definitions and conceptual change in mathematical practice, that is, the idea that mathematical proof may involve conceptual creativity: "the proof creates a new concept" (Wittgenstein, RFM III-41). Prover and Skeptic may in fact construe new concepts together in these dialogues, especially as various inferential relations are spelled out. The creative aspects of mathematical proof can be easily accommodated by the Prover-Skeptic model as long as the two characters continue to be sufficiently aligned in how they understand the key concepts at each stage. The fact that proofs create new concepts lends further support to the idea that there is nothing above and beyond the practices of deduction themselves that justifies deduction, as it is through and in these very practices that the conditions of applicability of (logical, mathematical) concepts are established. These observations in turn lead to a conception of definitions in mathematics as an essentially creative rather than descriptive endeavor.

Mireia Martínez i Sellarès (Utrecht University), **When are two geometrical figures affine? On a disagreement between Euler and Möbius**

Leonhard Euler introduced the notion of geometrical affinity in his Introductio in Analysin Infinitorum in 1748. Almost 80 years later, in 1827, August Möbius took up the same notion in his book Der Barycentrische Calcul. While crediting Euler with the definition of affinity, Möbius bluntly dismissed some of Euler's claims about the properties of this new concept. In fact, Möbius mistakenly believed that Euler's definition was the same as his own and, on this basis, he erroneously concluded that Euler had stated false theorems. However, Möbius was right in a certain way, as his definition was the "better" one. But what makes one definition "better" than another?

In analyzing this issue, we draw connections with current literature on the philosophy of mathematical practice. Our case study is an explicit case of a Prover-Skeptic interaction with a strength of disagreement that is unusual between two leading mathematicians. Besides, the dialogue between Euler and Möbius is also unusual in the sense that it takes place asynchronously.

Paul Hasselkuß (Heinrich Heine University Düsseldorf), **What's The Problem With Computer-Assisted Proofs?**

In the last decades, there has been a fiery debate about computer-assisted proofs (CAPs). Started by Appel & Haken's (1977) proof of the four-color theorem and the mixed reception it received, the debate centers around two issues. Let t be a mathematical theorem such that there is a CAP for t, but no ordinary (i.e. non-computational) proof. Firstly, is t merely verified by the CAP or does the CAP amount to a full proof (Rota 1997)? Secondly, is t known a posteriori because there is no proof for t that could be surveyed (Tymozko 1979, Kitcher 1984, McEvoy 2007)?

In the talk, I'll contribute to the debate in two ways. Firstly, I'll report initial findings from an ongoing data-driven study that aims to analyze articles published on the ArXiv to understand how, and to what extent, mathematicians rely on CAPs. Results may indicate whether the mathematical community takes CAPs to be on par with ordinary proofs or not.

Secondly, I'll argue that the justification provided by a CAP is different in degree, but not in kind, from the justification provided by an ordinary proof. If the latter is taken to yield a priori justification, the former does provide a priori justification, too. The argument relies on the idea that a CAP can be translated into an ordinary proof. While the translated proof is notoriously complex and probably impossible to survey by a single human agent, these differences are insufficient to claim that the CAP provides merely an a posteriori justification.

References

Appel, Kenneth, and Wolfgang Haken. 1977. "Every Planar Map is Four Colorable. Part I: Discharging." Illinois Journal of Mathematics 21 (3): 429–90.

Appel, Kenneth, Wolfgang Haken, and John Koch. 1977. "Every Planar Map is Four Colorable. Part II: Reducibility." Illinois journal of Mathematics 21 (3): 491–567.

Kitcher, Philip. 1984. The Nature of Mathematical Knowledge. New York: Oxford University Press. https://doi.org/10.1093/0195035410.001.0001

McEvoy, Mark. 2007. "The Epistemological Status of Computer-Assisted Proofs." Philosophia Mathematica 16 (3): 374–87. https://doi.org/10.1093/philmat/nkn014.

Rota, Gian-Carlo. 1997. "The Phenomenology of Mathematical Proof." Synthese 111 (2): 183–96. https://doi.org/10.1023/A:1004974521326.

Tymoczko, Thomas. 1979. "The Four-Color Problem and Its Philosophical Significance." The Journal of Philosophy 76 (2): 57–83. https://doi.org/10.2307/2025976.