$\mathbb{R}$ $\quad 2^\omega$

$X \subseteq 2^\omega$ is null $\iff \forall \varepsilon > 0 \; \exists (s_n)_{n\in\omega} \in 2^{<\omega}$

$\sum_{n=0}^{\infty} 2^{-|s_n|} < \varepsilon$

$X \subseteq \bigcup_{n\in\omega} [s_n]$.

SN

$X \subseteq 2^\omega$ is strong measure zero (smz) $\iff$

$\forall (\varepsilon_n)_{n\in\omega} \; \exists (s_n)_{n\in\omega}$

$\forall n \in \omega : |s_n| \geq \frac{1}{\varepsilon_n}$

$X \subseteq \bigcup_{n\in\omega} [s_n]$.

$|X| \leq \aleph_0 \Rightarrow X$ smz

smz σ-ideal

X perfect $\Rightarrow X \; \neg$smz

BC $:\iff (X \text{ smz} \Rightarrow |X| \leq \aleph_0)$

GMS $X \subseteq 2^\omega$ is smz $\Longleftrightarrow$ $\forall M \in \mathcal{M}$ $\boxed{\exists t \in 2^\omega \; (X+t) \cap M = \emptyset}$.

$\Updownarrow$

$\boxed{X + M \neq 2^\omega}$

$(2^\omega, +)$

$\mathcal{J} \subseteq \mathcal{P}(2^\omega)$ $\quad \mathcal{J}^* := \{X \subseteq 2^\omega : X + Z \neq 2^\omega \text{ for each } Z \in \mathcal{J}\}$

GMS: $\quad$ smz $= \mathcal{M}^*$

Definition (Prikry) $\quad$ sm (strongly meager) $:= \mathcal{N}^*$

$|X| \leq \aleph_0$ $\quad$ So $N \in \mathcal{N}$ $\quad X + N = \bigcup_{x \in X} \underbrace{x + N}_{\in \mathcal{N}} \in \mathcal{N}$

dBC $:\Longleftrightarrow$ sm $= [2^\omega]^{\leq \aleph_0}$.

$\mathcal{J}$-BC $:\Longleftrightarrow$ $\mathcal{J}^* = [2^\omega]^{\leq \aleph_0}$.

$$\begin{array}{ccc}
\mathcal{N}\text{-add.} & \Longrightarrow & \text{sm} \\
\Downarrow & & \Downarrow \;\not\Uparrow \\
\mathcal{M}\text{-add} & \Longrightarrow & \text{smz}
\end{array}$$

$X \subseteq 2^\omega$ is Marczewski null $:\iff \forall P$ perfect $\exists Q \le P$ perfect $Q \cap X = \emptyset$.

$(X \in s_0)$

$$\boxed{\forall p \in \mathbb{S} \ \exists q \le p \ \ [q] \cap X = \emptyset.}$$

$\mathbb{P}$ --- p_0

$$s_0\text{-BC} \iff s_0^* = [2^\omega]^{\le \aleph_0}.$$

$\Downarrow$

MBC

$\mathcal{M}^*$

$\mathcal{N}^*$

$$\aleph_1 \le \mathrm{add}(\mathcal{J}) \le \mathrm{non}(\mathcal{J}) \le \mathrm{cof}(\mathcal{J}) \le 2^{\aleph_0}$$

$$\mathrm{add}(\mathcal{J}) \le \mathrm{cov}(\mathcal{J}) \le \mathrm{cof}(\mathcal{J})$$

$$\mathrm{ZFC} \vdash \mathrm{cof}(s_0) > 2^{\aleph_0}.$$

$\forall A \in \mathcal{J} \ \exists B \in \mathcal{B}_{\mathrm{asis}} \ A \subseteq B.$

$\mathrm{cov}(\mathcal{J}) \ge \mathrm{cof}(\mathcal{J}) =: \kappa$

$\Rightarrow \exists |X| = \kappa \ \ X \in \mathcal{J}^*.$

L_0 m_0

Definition: $\mathcal{J} \subseteq \mathcal{P}(2^\omega)$ is Sacks dense ideal (S.d.I.)

$\Longleftrightarrow$ (1) $\mathcal{J}$ is σ-ideal

(2) $\mathcal{J}$ is translation-invariant $\longleftarrow$

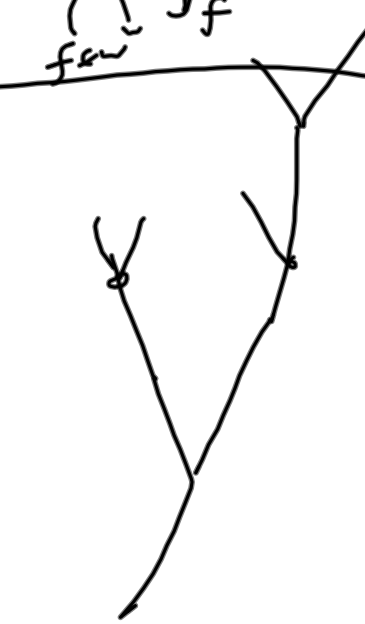
(3) $\forall p \in \mathbb{S} \quad \exists q \le p \quad [q] \in \mathcal{J}$.

Lemma: (CH) $\mathcal{J}$ S.d.I. $\Rightarrow s_0^* \subseteq \mathcal{J}$.

$$\underset{\text{NBC}}{\underset{?}{[2^\omega]^{\le\aleph_0}}} \subseteq s_0^* \subseteq \underbrace{\bigcap_{\mathcal{J} \text{ S.d.I.}} \mathcal{J}}_{\bigcap_{f\in\omega^\omega} \mathcal{J}_f} \subsetneq \mathcal{M} \cap \mathcal{N}$$

$f \in \omega^{\omega\uparrow}$ $\quad \mathcal{J}_f := \sigma\langle\{P \subseteq 2^\omega \mid P \text{ } f\text{-tiny}\}\rangle$

$$|P \restriction f(n)| \le n$$



ZFC $\vdash$ $\bigcap_{f \in \omega^\omega} \mathcal{J}_f \subseteq \mathcal{N}$-add. $\subseteq$ smz

$\cup$

s_0^*

Lemma: $s_0^* \subseteq \mathcal{J}$

ωMBC $:\longleftrightarrow$ $s_0^* \subseteq [2^\omega]^{<2^{\aleph_0}}$.

CH Question: $\mathcal{R} := \bigcap_{\mathcal{J} \text{ s.d.I.}} \mathcal{J} = [2^\omega]^{\leq \aleph_0}$.

Σ^2_1

Π^2_1

CH $(\mathcal{J}_\alpha)_{\alpha<\omega_1}$ s.d.I. $\Rightarrow \exists X, |X| = \aleph_1$

$X \in \bigcap_{\alpha<\omega_1} \mathcal{J}_\alpha$.

$\wedge \exists \mathcal{J}'$ s.d.I. $X \notin \mathcal{J}'$.