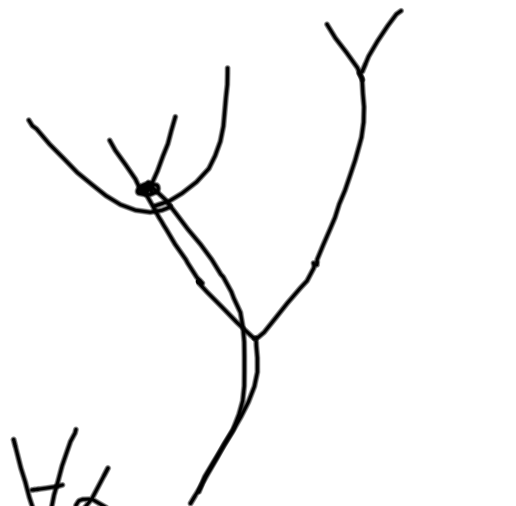
$T \in \mathbb{P}$ $\quad T \subseteq 2^{<\omega}$ $\qquad S \leq T :\iff S \subseteq T \iff [S] \subseteq [T]$

$T \subseteq \omega^{<\omega}$

$[T] = \{x : \forall n \; x \restriction n \in T\}$

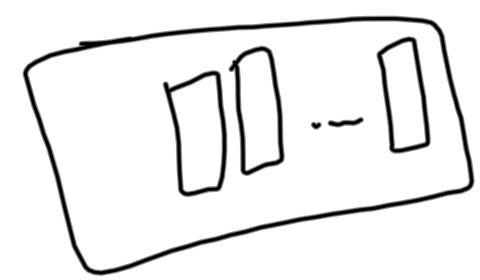


•) $T \in \mathbb{P},\ t \in T \Rightarrow T \restriction t \in \mathbb{P}$

$T \restriction t = \{s \in T : s \leq t \vee t \leq s\}$

•) $T \in \mathbb{P} \Rightarrow \exists \{T_\alpha : \alpha < 2^{\aleph_0}\}$ mit $T_\alpha \leq T \;\; \forall \alpha$

$\forall \alpha \neq \beta : [T_\alpha] \cap [T_\beta] = \emptyset \;\; (\Rightarrow T_\alpha \perp T_\beta)$



(insbes.: $\mathbb{P}$ ist nicht ccc)

•) $\mathbb{P}$ is homogeneous

$\mathbb{P} \xhookrightarrow{d} \mathrm{Borel}/\mathcal{J}_{\mathbb{P}}$ $\qquad$ $\mathbb{S}$ ... $\mathcal{J}_{\mathbb{S}} = \{\text{countable sets}\}$

$\forall B$ Borel $B \notin \mathcal{J}_{\mathbb{P}}$ $\exists T \in \mathbb{P} : [T] \subseteq B$; $T \in \mathbb{P}$ $[T] \notin \mathcal{J}_{\mathbb{P}}$

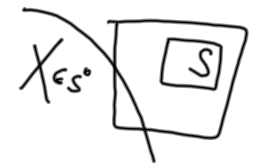
$S, T \in \mathbb{P}$ $\quad S \perp T \iff [S] \cap [T] \in \mathcal{J}_{\mathbb{P}}$.

$\mathcal{J}_{\mathbb{L}} = \{ X \subseteq \omega^\omega \mid X$ nicht strongly dominating $\}$

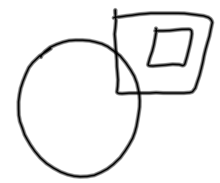
$\mathcal{J}_{\mathbb{M}} = \{ X \subseteq \omega^\omega \mid X$ K_σ, X bounded $\}$

$\mathbb{P}$ $\quad p^\circ := \{X \subseteq 2^{\omega^\omega} : \forall T \in \mathbb{P}\ \exists S \le T : [S] \cap X = \emptyset\}$.

$\mathbb{S}$ $\quad s^\circ$



$\mathcal{J}_\mathbb{P} \subseteq p^\circ$ $\quad X \in \mathcal{J}_\mathbb{P}$



$\mathcal{J}_\mathbb{S}$

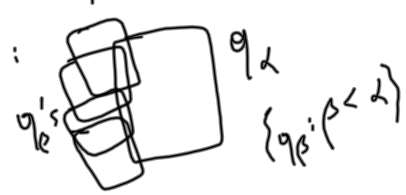
Fact: $\mathcal{J}_\mathbb{S} \subsetneq s^\circ$.

Proof: Wähle max. AK $A = \{q_\alpha : \alpha < 2^{\aleph_0}\} \subseteq \mathbb{S}$.

$$\forall \alpha \ne \beta \quad q_\alpha \perp q_\beta \quad (|[q_\alpha] \cap [q_\beta]| \le \aleph_0)$$

$$\forall p \in \mathbb{S} \quad \exists \alpha < 2^{\aleph_0} \quad p \not\perp q_\alpha$$

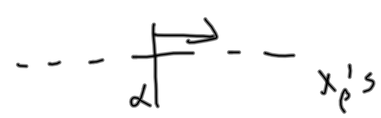
$X := \{x_\alpha : \alpha < 2^{\aleph_0}\}$. Bei Schritt α:

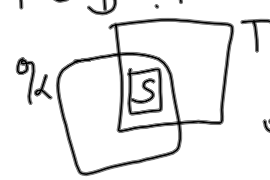


$x_\alpha \in [q_\alpha] \setminus \bigcup_{\beta<\alpha} [q_\beta]$.

Behauptung: $X \in s^\circ$. $\quad T \in \mathbb{S}$. Fixiere $\alpha < 2^{\aleph_0}$: $T \not\perp q_\alpha$

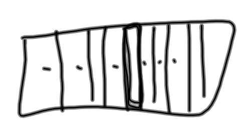
$$\begin{array}{ccc} T & \not\perp & q_\alpha \\ \rotatebox{90}{$\le$} & & \rotatebox{90}{$\le$} \\ & S & \end{array}$$





alle x_β mit $\beta > \alpha$, $x_\beta \notin [S]$.

$$\begin{array}{l} X \cap [S] \subseteq \{x_\beta : \beta \le \alpha\} \\ \quad \rotatebox{90}{$\le$} \\ \quad S' \in \mathbb{S} \qquad [S'] \cap X = \emptyset, \text{ also } X \in s^\circ. \end{array}$$



$\mathbb{P}$ $\quad p^\circ \cap \text{Borel} =: \mathcal{J}_\mathbb{P}$

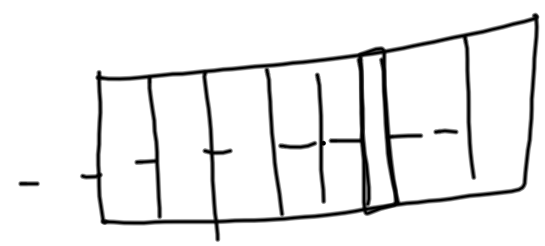
$$\operatorname{non}(\mathcal{I}) = \min\{|A| : A \notin \mathcal{I}\}$$

$$\operatorname{cof}(\mathcal{I}) = \min\{|\mathcal{B}| : \underbrace{\mathcal{B} \subseteq \mathcal{I} \text{ Basis}}_{\Updownarrow \; \forall A \in \mathcal{I} \; \exists B \in \mathcal{B} : A \subseteq B.}\}$$

Fact $\operatorname{non}(\mathcal{I}) \leq \operatorname{cof}(\mathcal{I})$.

$\mathbb{P}$ $\quad \operatorname{non}(p^0) = 2^{\aleph_0}$. $\quad |X| < 2^{\aleph_0} \Rightarrow X \in p^0$.

$T \in \mathbb{P}$



$$\Rightarrow \quad 2^{\aleph_0} = \operatorname{non}(p^0) \leq \operatorname{cof}(p^0) \leq 2^{(2^{\aleph_0})}.$$

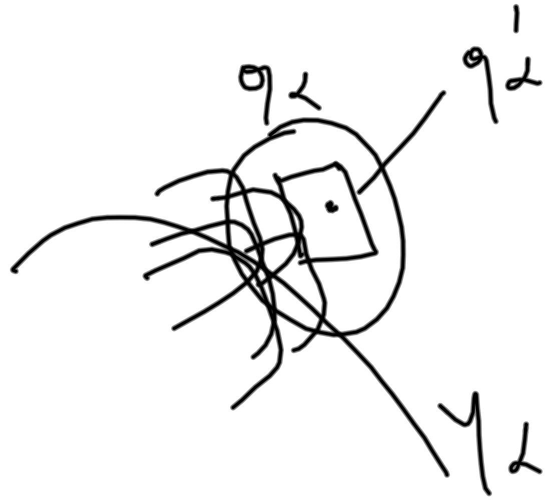
$\mathbb{S}$: $\quad \operatorname{cof}(s^0) > 2^{\aleph_0}$.

$\mathbb{P}$: $\quad \boxed{\operatorname{cof}(p^0) > 2^{\aleph_0} ?}$

Theorem: $\operatorname{cof}(s^0) > 2^{\aleph_0}$.

Proof: A max. AK, $|A| = 2^{\aleph_0}$.

Ang. nicht: $\{Y_\alpha : \alpha < 2^{\aleph_0}\} \subseteq s^0$. Gesucht: $X \in s^0$ mit $X \not\subseteq Y_\alpha \;\forall \alpha$



$[q'_\alpha] \cap Y_\alpha = \emptyset$

x_α

Theorem: $\operatorname{cov}(\mathcal{J}_p) = 2^{\aleph_0} \Longrightarrow \operatorname{cof}(p^0) > 2^{\aleph_0}$.

$\mathbb{S}$: **Fact:** Es gibt eine disjunkte maximale AK der Größe $2^{\aleph_0}$ in $\mathbb{S}$.

$$\mathrm{add}(\mathcal{J}_{\mathbb{P}}) = 2^{\aleph_0}$$

$$\mathrm{add}(\mathcal{J}_{\mathbb{P}}) \leq \underset{\overset{\|}{2^{\aleph_0}}}{\mathrm{add}(\mathcal{J}_{\mathbb{P}}, p^\circ)} \leq \mathrm{cov}(\mathcal{J}_{\mathbb{P}})$$

Theorem: A disjunkte maximale AK $\subseteq \mathbb{P}$ der Größe $2^{\aleph_0}$

$$\Longrightarrow \mathrm{cf}(\mathrm{cof}(p^\circ)) > 2^{\aleph_0}$$

$$\Longrightarrow \mathrm{ZFC} \vdash \mathrm{cf}(\mathrm{cof}(s^\circ)) > 2^{\aleph_0}$$

r°

v°

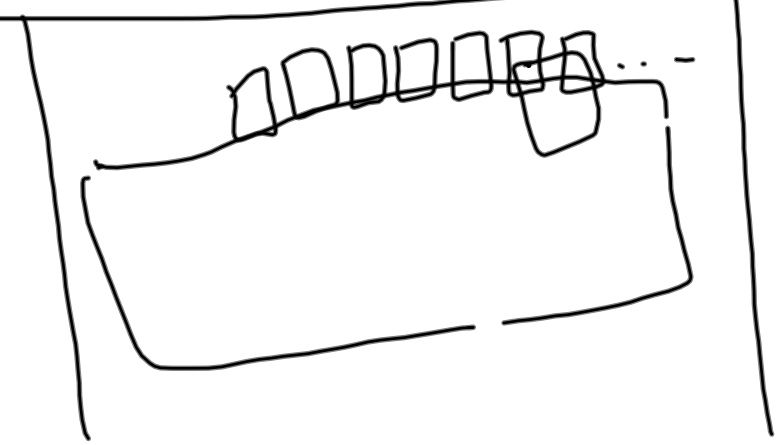
$A \subseteq \mathbb{P}$ ist eine Brendle-Antikette: ·) Größe $2^{\aleph_0}$

·) disjunkt: $\forall p \neq q \in A$
$[p] \cap [q] = \emptyset$.

·) $\forall T \in \mathbb{P}$: <u>either</u> $T \not\perp p$ für ein $p \in A$

<u>or</u> $\exists S \leq T$:
$\forall p \in A : \big|[S] \cap [p]\big| \leq 1$.

$ZFC \vdash \exists$ Brendle-AK in $\mathbb{L}$.



Wenn A eine Brendle-AK in $\mathbb{P}$ $\Longrightarrow$ $cf(cof(p^\circ)) > 2^{\aleph_0}$.