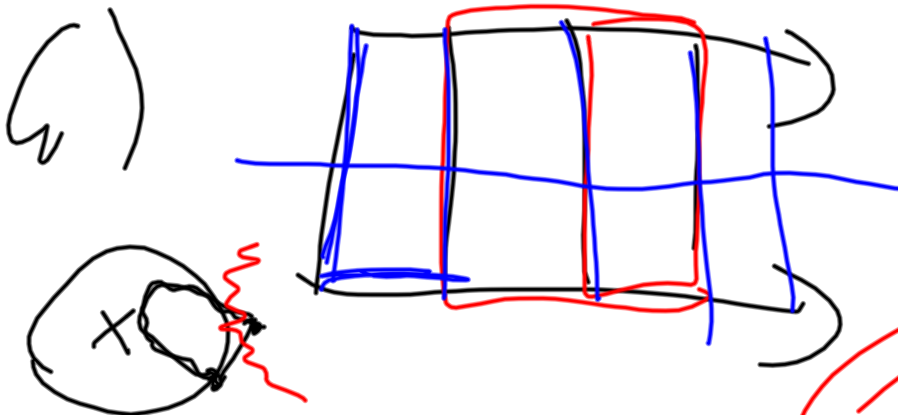
What is a tame matroid

Ex. $M_F(G)$

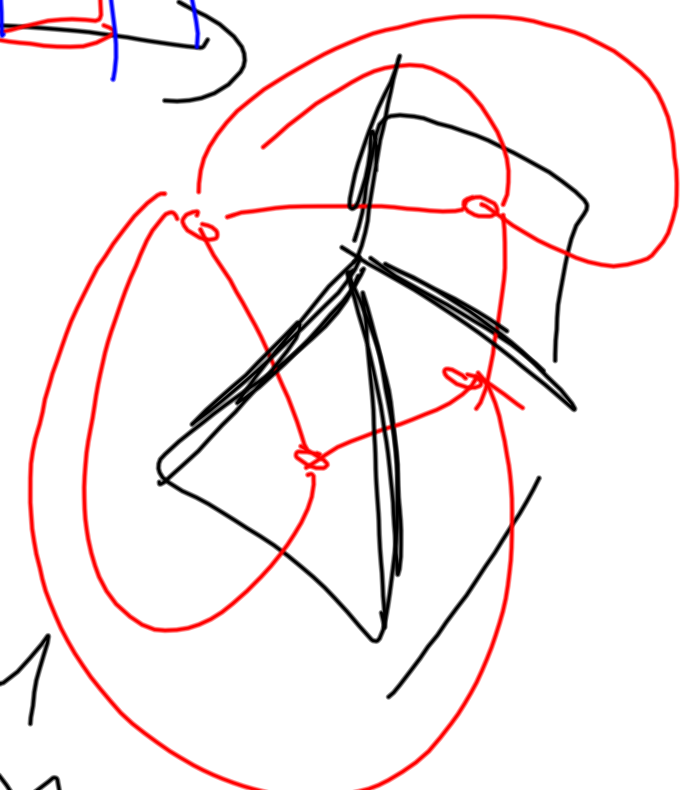


Axioms: $\mathcal{C}, \mathcal{D} \subseteq \mathcal{P}(E)$

E countable.

$\mathcal{C}, \mathcal{D}$, $\mathcal{C}, \mathcal{D}$ antichains

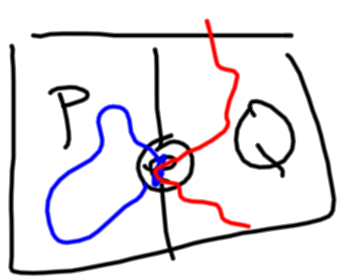
$\emptyset \notin \mathcal{C}, \emptyset \notin \mathcal{D}$



(O1) $\forall o \in \mathcal{C}, \forall d \in \mathcal{D}$: $|o \cap d| \neq 1$

(tameness) $|o \cap d| \neq \infty$

(O2) $\forall E = P \dot\cup Q \dot\cup \{e\}$



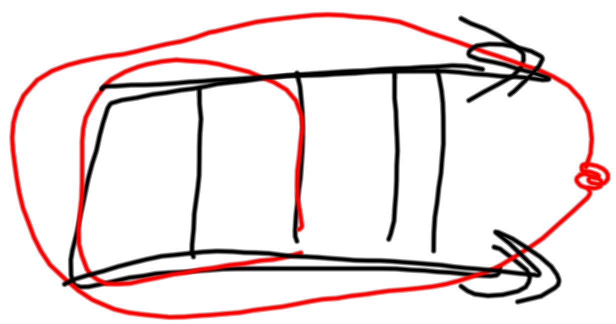
either $\exists o \in \mathcal{C}$ with $e \in o \subseteq P+e$ or

$\exists d \in \mathcal{D}$ with $e \in d \subseteq Q+e$

Basic examples: G-matroids

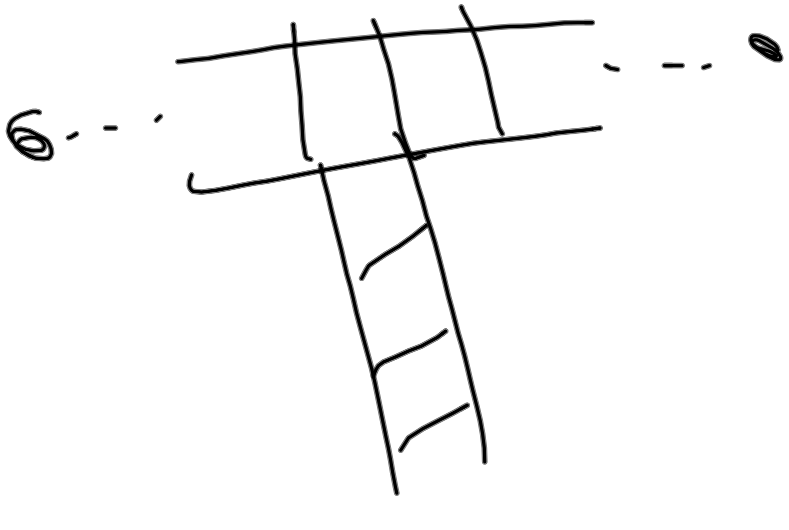
G loc. fin.

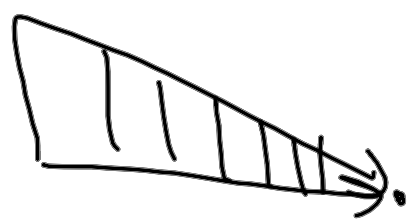
Ex 2:

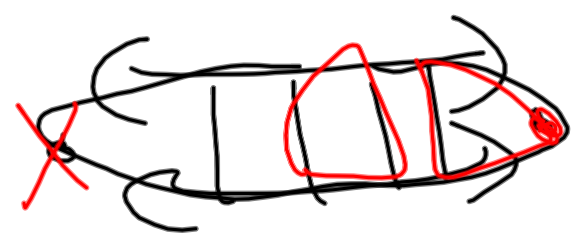


$$M_{FC}(G)^* = M_B(G)$$

$$M_{TC}(G)^* = M_{FB}(G)$$

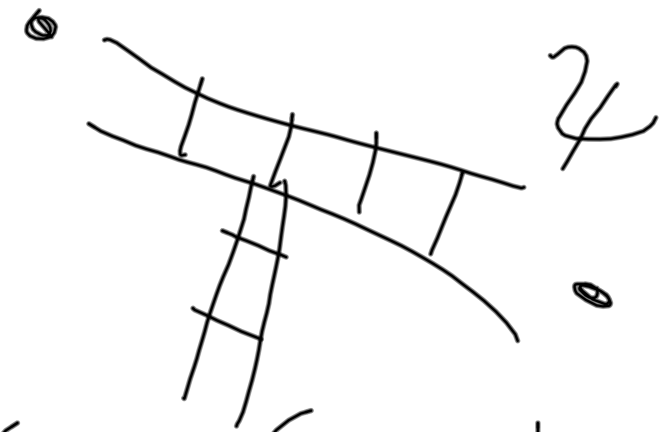






A tame matroid M is a G-matroid if all its circuits are top. circles of G
AND " cocircuits are bonds of G.

Ψ-"matroid":



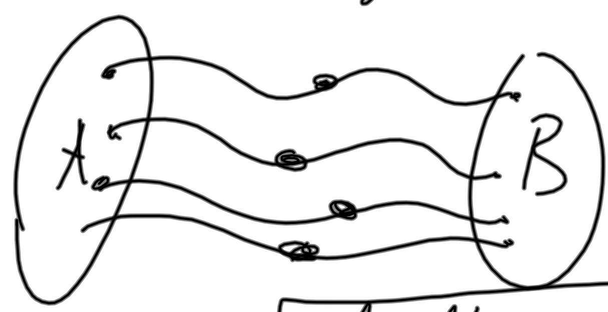
<u>Thm (BC)</u>: Every G-matroid is a Ψ-matroid of G.

$\exists\, G, \Psi \subseteq \Omega(G)$ s.th. the Ψ-"matroid" is <u>not</u> a matroid.

Thm (BC): If Ψ is Borel, then every Ψ-matroid is a matroid.

P/C-conj:

Thm (Aharoni, Berger '08): $\forall$ G graph, $\forall A, B \subseteq V(G)$

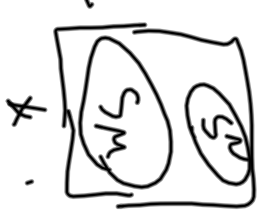


M, N matroids on the same groundset E.

$X \overset{\subseteq E}{}$ is a **packing** if $\exists\, S^M, S^N$ disjoint, spanning in $M\restriction_X, N\restriction_X$, resp.

Y is a **covering** if it is a packing for M^*, N^*.

X



P/C-conj: $\forall M, N$ on G.

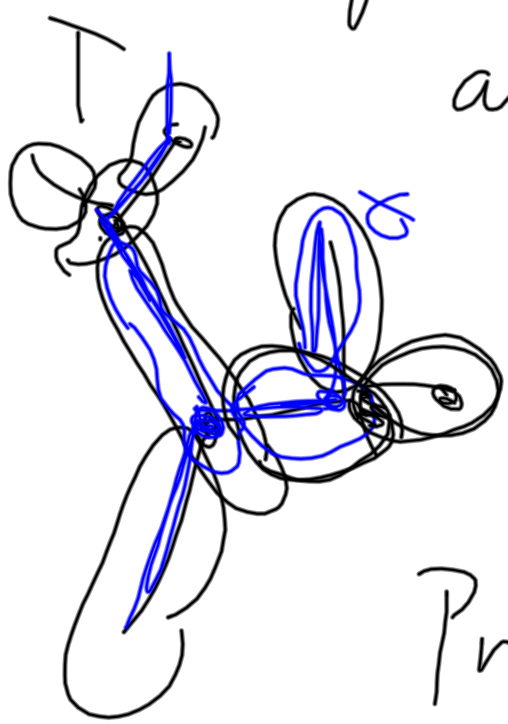
$\exists$ a partition $E = P \mathbin{\dot\cup} C$

Packing

covering.

tree of binary matroids

A **tree of binary matroids** is a pair (T, M) where T is a tree and M assigns a binary matroid $M(t)$ to each $t \in V(T)$ s.t.

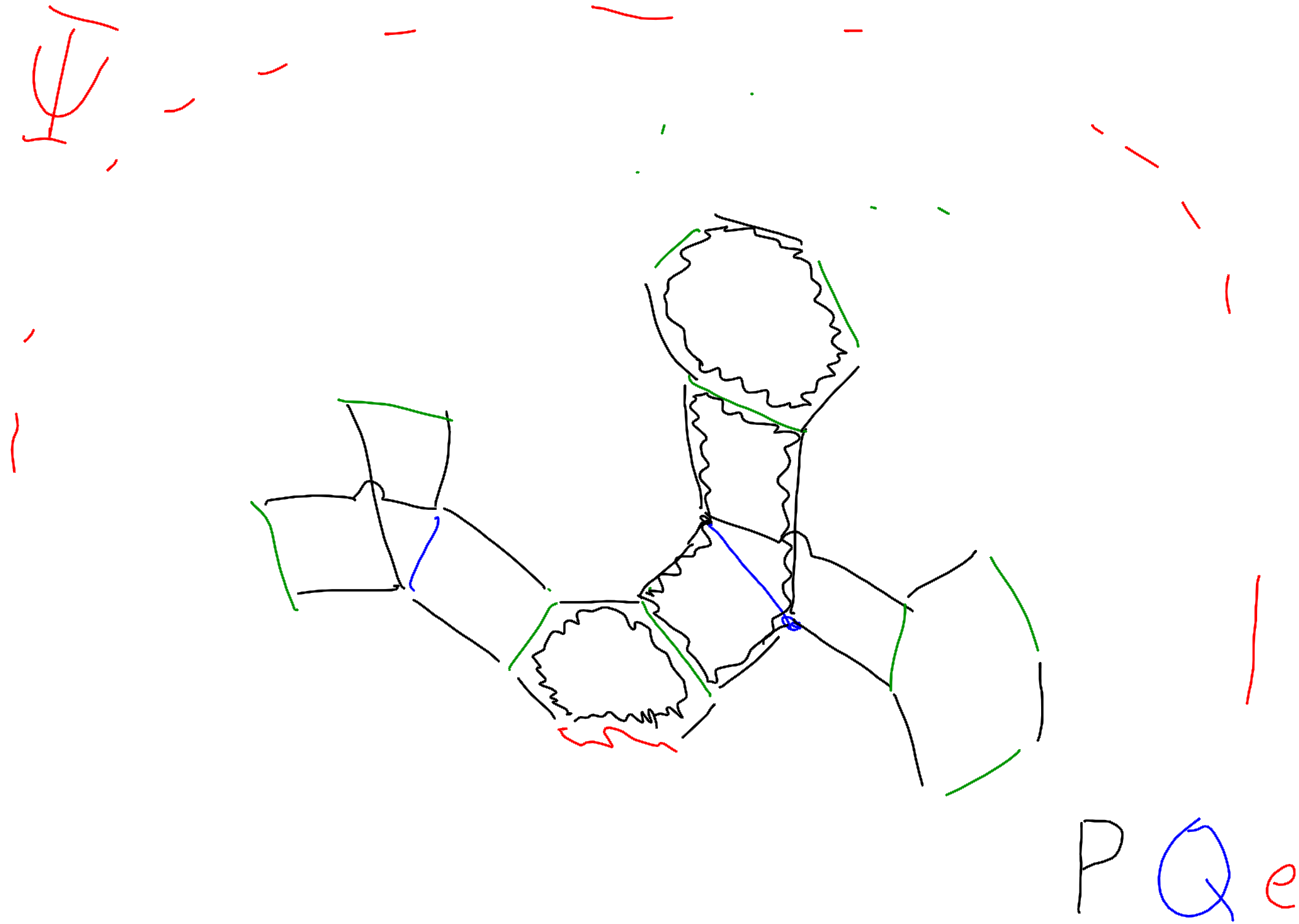


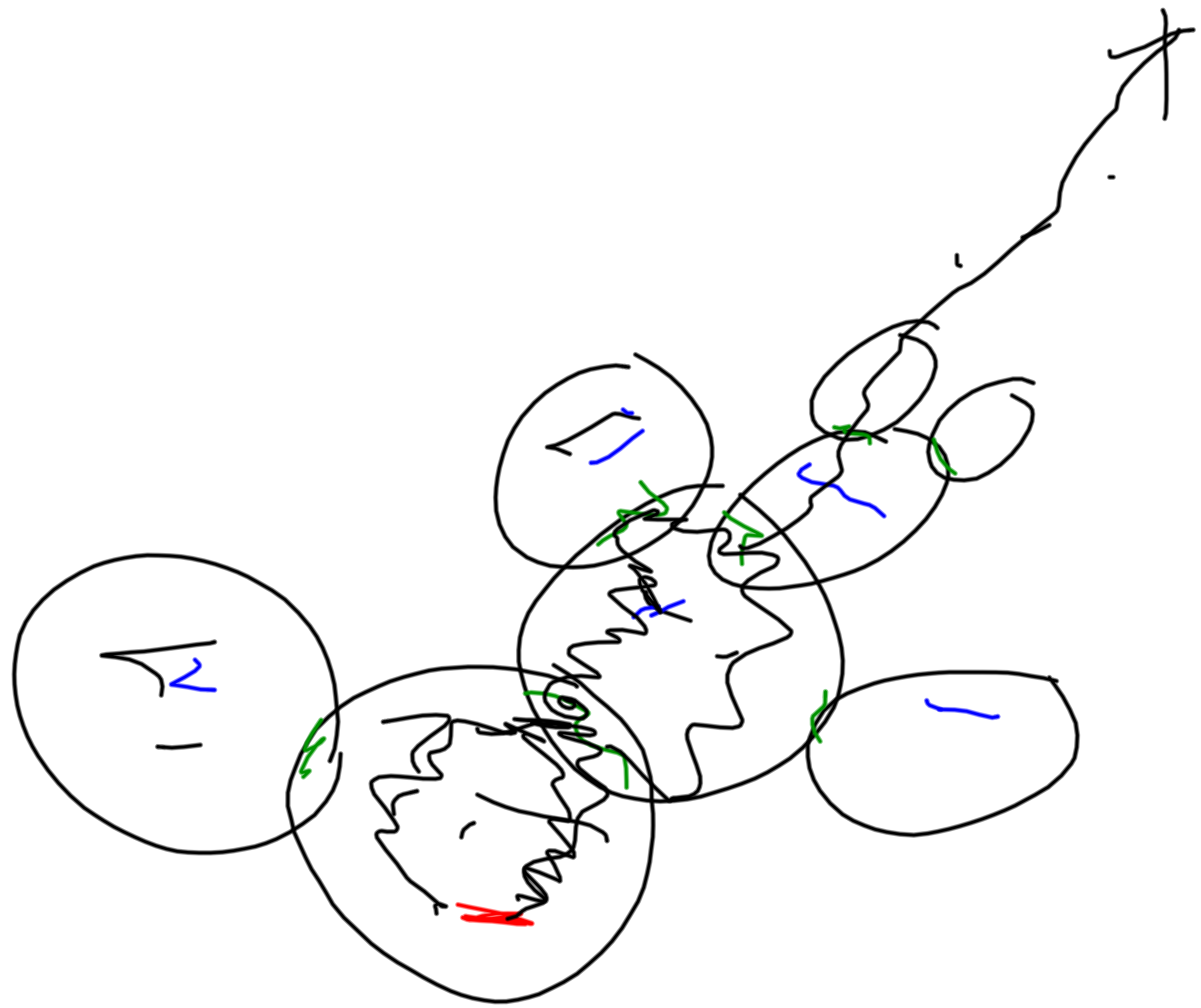
$$E(M(t)) \cap E(M(t')) \neq \emptyset$$
iff $tt' \in E(T)$.

Precircuit.

Σ of edges. $\rightarrow \Psi$-Precircuit

Ψ-circuit.


Ψ
P Q e



Let (T, M) and (T, N) be trees of matroids of overlap 1, with $E(M(t)) = E(N(t))$ for each $t \in V(T)$. Let $\Psi_M, \Psi_N \subseteq \Omega(T)$. Suppose $M_{\Psi_M}(T, M)$ and $M_{\Psi_N}(T, N)$ are matroids. Do they satisfy P/C?

Lem: for any M, N there is a maximal (M,N)-packing.

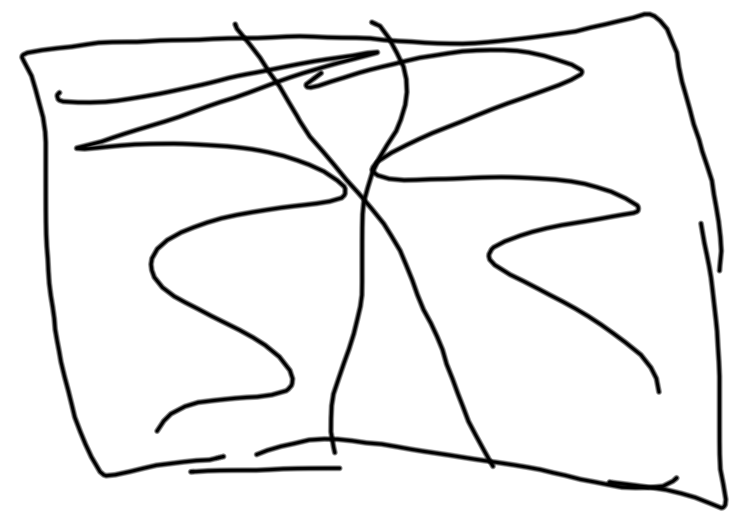
Sim for coverings

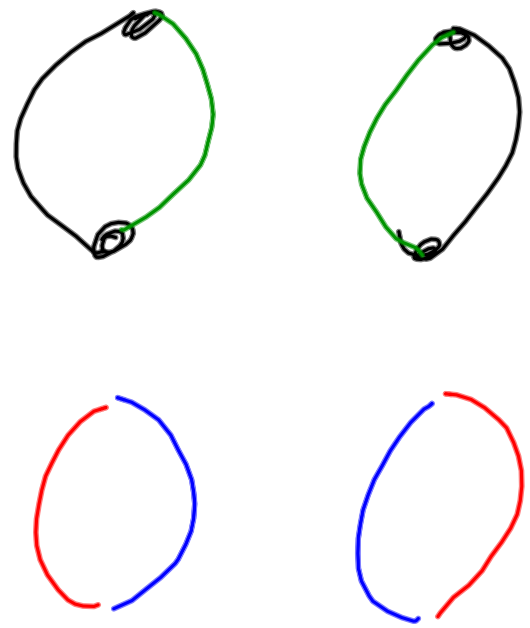
So P/C is equivalent to:

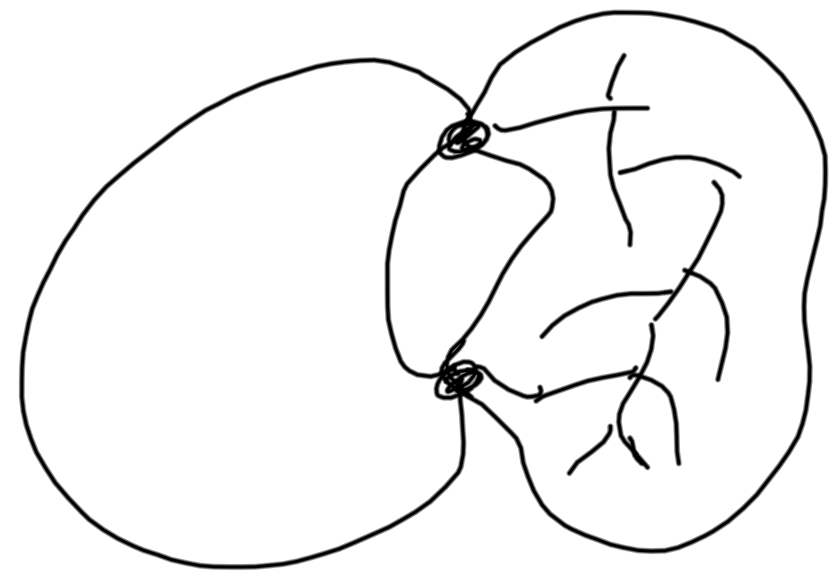
$\forall M, N, \forall e \in E$, one of:

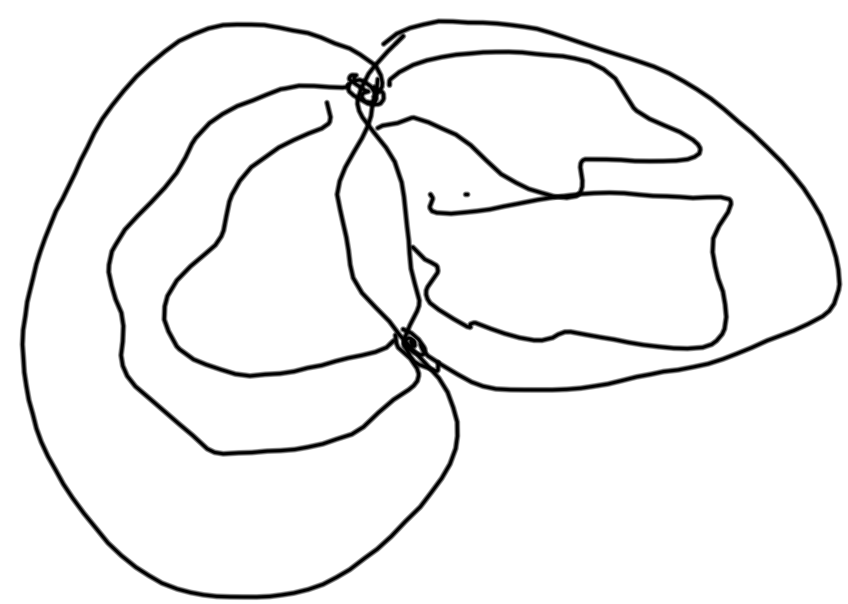
$\exists$ packing containing e

$\exists$ covering containing e









The 6 Promises:

Span in both matroids

Span in M

Span in N

Span in M provided that spanned in N

Span in N provided spanned in M

No promise

The packing game (for a promise P):