

Übung 4 zur Graphentheorie

Ihre Lösung zu der mit * markierten Aufgabe geben Sie bitte am Anfang Ihrer jeweiligen Übungsgruppe ab. Ihre Lösungen zu den anderen Aufgaben bringen Sie bitte präsentierbereit zur Übung mit.

1. Moving alternately, two players jointly construct a path in some fixed graph G . If $v_1 \dots v_n$ is the path constructed so far, the player to move next has to find a vertex v_{n+1} such that $v_1 \dots v_{n+1}$ is again a path. Whichever player cannot move loses. For which graphs G does the first player have a winning strategy, for which the second?
2. Derive the marriage theorem from König's theorem.
3. Show that if there exist injective functions $A \rightarrow B$ and $B \rightarrow A$ between two infinite sets A and B then there exists a bijection $A \rightarrow B$.
- 4.* Let k be an integer. Show that any two partitions of a finite set into k -sets admit a common choice of representatives.
5. Show that all stable matchings of a given graph cover the same vertices. (In particular, they have the same size.)

Hinweise

1. Consider a maximal matching in G .
2. If there is no matching of A , then by König's theorem few vertices cover all the edges. How can this assumption help you to find a large subset of A with few neighbours?
3. Turn the functions into a multigraph, and consider its components.
- 4.* Apply the marriage theorem.
5. Alternating paths.