

Übung 3 zur Graphentheorie

Ihre Lösung zu der mit * markierten Aufgabe geben Sie bitte am Anfang Ihrer jeweiligen Übungsgruppe ab. Ihre Lösungen zu den anderen Aufgaben bringen Sie bitte präsentierbereit zur Übung mit.

1. Show that every automorphism of a tree fixes a vertex or an edge.
2. Revisit the proof that every connected graph has a spanning tree given as an application of Theorem 1.5.1 (iii). What is wrong with the analogous ‘proof’ that says, ‘take a maximal acyclic subgraph and apply (iv)’? (Hint: Something must be wrong, as the ‘proof’ does not use the assumption that the graph is connected. But where exactly is the error?)
3. Show that every path in a finite connected graph extends to a normal spanning tree.
- 4.* Show that a graph is 2-edge-connected if and only if it has a *strongly connected* orientation, one in which every vertex can be reached from every other vertex by a directed path.
5. Is the vertex partition of a bipartite graph uniquely determined?

Hinweise

1. Induction on $|T|$ is a possibility, but not the only one.
2. What does 'maximal' mean?
3. The assertion can be proved by modifying a proof from the lecture. But there are at least two other, very different, induction proofs; who can find one of those?
- 4.* Use a normal spanning tree.
5. Spanning trees.