Symmetric spaces and vector bundles

Abstract:

Locally, Riemannian geometry is just almost euclidean geometry with all its features. The deviation is measured by the famous curvature tensor introduced by Riemann. Consequently, the most basic Riemannian spaces are those where the curvature tensor is constant (parallel): (locally) symmetric spaces. Among them are compact Lie groups, spheres, Grassmannians (sets of linear subspaces) and sets of real, complex or quaternionic structures on vector spaces.

These spaces have many applications outside Riemannian geometry. One of them concerns K-theory, the theory of real, complex or quaternionic vector bundles over a compact manifold X. A vector bundle is a family of vector spaces E_x depending continuously on a parameter $x\in X$. A fundamental fact in this area is the periodicity theorem of Bott and Atiyah. For vector bundles over the reals it asserts that the (real) K-rings for X and $X \times S^8$ are essentially the same. The isomorphism is obtained by tensorizing the given vector bundle over X with the Hopf vector bundle over the 8-sphere S^8 which just encodes the multiplicative structure of the octonionions, the non-associative brothers of real, complex and quaternionic numbers. This classical theorem can be reproved and generalized using old ideas of Bott and Milnor, related to a purely geometric construction of "equators" in compact symmetric spaces (common work with Bernhard Hanke).

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