

Matrix Spaces vs. Projective Lines over Rings

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L. K. Hua and W. L. Chow initiated what today is known under the name “geometry of matrices” around the year 1945. Here one is concerned with spaces whose points arise in some way from matrices over a field. The point set of such a *matrix space* is endowed with a binary relation, usually called *adjacency*. It turns the matrix space into an undirected graph. The problem of finding all automorphisms of such graphs has attracted many authors. Solutions to this problem are usually stated as a fundamental theorem for a geometry of matrices.

The roots of ring geometry are not so easy to ascertain. It seems though that W. Blaschke and C. Segre were among the first to exhibit geometries with coordinates coming from a proper ring. This was at the beginning of the 20th century. We focus our attention to *projective lines over a ring*. There is a binary relation, called the *distant relation*, on the point set of such a line. Hence, like before, we may exhibit the problem of describing all automorphisms of the graph arising from the line’s distant relation. In general, a “nice” characterisation of these mappings seems hopeless, but for certain classes of rings a description in the spirit of matrix spaces is possible. The aim of our lecture is to give a survey on the known links between these two mathematical disciplines which, for a long period of time, have evolved independently from each other.