

Homogeneous Lorentzian manifolds

D. V. Alekseevsky

Abstract

We discuss the problem of description of homogeneous Lorentzian manifolds $M = G/H$ with proper and nonproper action of an isometry group G .

We recall the classical result by Nadine Kowalsky about nonproper action of a simple isometry group on a Lorentzian manifolds and its recent generalizations.

We state a necessary and sufficient condition that a proper homogeneous manifold $M = G/H$ (i.e. a homogeneous G -manifold with a proper action of G) admits an invariant Lorentz metric.

We give a description of minimal compact homogeneous Lorentzian G -manifolds $M = G/H$ (that is a Lorentzian manifold $M = G/H$ such that any homogeneous G -manifold with bigger stability group $H' \supset H$ does not admit an invariant Lorentzian metric).

In the case of a noncompact semisimple group G , we reduce the description of proper homogeneous Lorentzian G -manifolds $M = G/H$ to determination of the stability subgroups of the isotropy representation of the corresponding noncompact symmetric space $S = G/K$.

We give a list of all homogeneous Lorentzian G -manifolds $M = G/H$ of dimension $m \leq 11$, where G is a noncompact simple Lie group.

At the end, we discuss the problem of description of non proper homogeneous Lorentzian manifolds of non semisimple Lie group G and give a classification of such manifolds with irreducible action of the isotropy group on the screen space.