<u>Embedding of parallelisms in projective spaces</u>

Clifford parallelisms in the real projective space $\mathbb{RP}^3$ can be defined by means of various different geometric tools, even with no need of metric nor differential concepts, namely either considering the family of line spreads that are in fact the *elliptic congruences* determined by each of the two reguli of an imaginary (anisotropic) quadric of $\mathbb{RP}^3$, or regarding, on the Klein quadric of $\mathbb{RP}^3$, two suitable families of *ovoids* cut out by two 3-space pencils with mutually polar planes as axes. They can also be characterised in a purely algebraic way using left and right multiplications in the Hamilton quaternion division algebra $\mathbb{H}(\mathbb{R})$. Such constructions make sense in any pappian projective 3-space, whenever an anisotropic 4-dimensional quadratic form does exist; in particular, if the ground field $\mathbb{F}$ admits more than one quadratic extension, the discussion leads to providing a geometric characterization of those quadratic extensions of $\mathbb{F}$ which are contained in a fixed quaternion skew field over $\mathbb{F}$. Moreover new “non-Clifford” regular parallelisms can be obtained, using a method which has no corresponding in the classical case.
As a further generalization, we consider and discuss examples of *translation spaces with parallelism* (*P,L*) embedded in a projective space Σ in such a way that the point set *P* is a subset of the point set of the projective space, the lines of *L* are (proper or improper) subsets of suitable projective subspaces of Σ and the translation group is a subgroup of the whole collineation group of the projective space itself.

Prof. Dr. Silvia Pianta (Università Cattolica di Brescia)