Amenability, self-similarity and entropy

We shall discuss a new development at the crossroads of Analysis, Algebra and Probability. Amenability (its definition going back to von Neumann) is, from the analytical point of view, the most natural generalization of finiteness or compactness. Namely, amenable groups are those which admit an invariant mean (rather than an invariant probability measure, which is the case for finite or compact groups). Self-similar groups (automata groups, iterated monodromy groups, etc.) have recently become the object of an extensive study in the group theory, since even in the simplest situations such groups may have rather unusual properties. We shall describe a new technique for proving amenability of self-similar groups ("Münchhausen trick") developed by the author and based on using the notion of the asymptotic entropy of a random walk. This technique has recently led to a proof of amenability for a large class of self-similar groups by Bartholdi, Nekrashevych, Virag and the author.

Prof. Dr. Vadim Kaimanovich (Jacobs University Bremen)