This talk will focus on several aspects of the theory of arithmetic groups with an emphasis on the relations among geometric aspects of the corresponding locally symmetric spaces and questions in the arithmetic of algebraic groups. We will illustrate the richness of methods and results in this area of research by describing various geometric constructions of non-vanishing cohomology classes for arithmetic groups. The class of locally symmetric spaces that are quotients of hyperbolic $n$-space will serve as an example. This includes arithmetically defined hyperbolic $3$-manifolds as well as classical arithmetic quotients that arise via lattices coming from quadratic forms over algebraic number fields. Concluding remarks will describe open questions, in particular, some which pertain to the relation between the cohomology of arithmetic groups and the theory of automorphic forms.