It’s all about power: definition, modeling and control of swarm type direct current microgrids

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Talk, Workshop on Industrial and Applied Mathematics 2016
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Outline

1. Introduction
2. Modeling and simulation frameworks
3. Possible interfaces to applied mathematics
4. Modeling
5. Control
6. Ongoing and future work
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Solar Home Systems in Bangladesh

**Figure:** Solar Home System (SHS) (left); SHSs in Raipura, Bangladesh (right)
Definition

An electrical energy provision system is called **Solar Home System (SHS)** if it satisfies the following conditions.

1. **It is composed by a photovoltaic (PV) panel, a battery storage, a charge controller and domestic loads.**
2. **The voltage level is 12-220 volt direct current (DC), where 12 volt is more common.**
3. **It usually operates independently as islanded system. It can be connected to other solar home systems through the swarm concept.**
The swarm type low voltage direct current microgrid

Swarm concept [Str15]

Definition

The **swarm concept** is a bottom-up electrification scheme which

1. interconnects existing generation, storage and consumption units to form an electrical power grid, typically where a cluster of stand-alone energy provision systems is installed, e.g., SHSs. This setting is called swarm type cluster;

2. allows for plug-and-play operation, i.e., each unit can connect to or disconnect from the grid;

3. grows organically, i.e., the network topology changes [...] arise spontaneously when a new household is connected. It interconnects swarm type clusters with each other or with existing true islanded power systems called microgrids, minigrids or nanogrids in practice through a point of common coupling. This setting is called swarm type microgrid;

4. grows towards and eventually reaches the main power grid, which is usually operated by a national power entity, in order to draw or feed-in power.
The swarm type low voltage direct current microgrid

PhD thesis (objective):

How can a swarm type low voltage DC microgrid be modularly modeled, controlled and simulated in order to contribute to the technology development?

→ I am in the first year comparing different modeling and simulation frameworks for control and did some preliminary work in my master thesis
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# Modeling frameworks, qualitative comparison

<table>
<thead>
<tr>
<th>Framework Criteria</th>
<th>ODEs</th>
<th>DAEs</th>
<th>DESs</th>
<th>HS</th>
<th>BG</th>
<th>pH</th>
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<td>Pre-knowledge of the author</td>
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<td>high</td>
<td>medium</td>
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<td>low</td>
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<td>Access to gurus</td>
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<tr>
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<td>advanc.</td>
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<tr>
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<td>high</td>
<td>increas.</td>
<td>?</td>
<td>?</td>
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<tr>
<td>Analytical handling</td>
<td>easy</td>
<td>ok for s-free</td>
<td>diffic.</td>
<td>diffic.</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Numerical handling</td>
<td>easy</td>
<td>index dep.</td>
<td>easy</td>
<td>diffic.</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Bidirectional power flow</td>
<td>diffic.</td>
<td>diffic.</td>
<td>easy</td>
<td>easy</td>
<td>easy?</td>
<td>easy</td>
</tr>
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</table>

### Simulation frameworks

<table>
<thead>
<tr>
<th>Programm</th>
<th>Dymola</th>
<th>Matlab/Simulink</th>
<th>Octave</th>
<th>Scilab/Xcos, PowerDevs</th>
<th>20-sim</th>
<th>Phyton</th>
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<tbody>
<tr>
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<tr>
<td>Cost [Euro]</td>
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<td>&gt;&gt; 2000</td>
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<td>&gt; 1000</td>
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<tr>
<td>Open Source</td>
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<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Not anymore</td>
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<td>Platform</td>
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<td>L</td>
<td>L,W</td>
<td>W,M,L?</td>
</tr>
</tbody>
</table>

* language Modelica yes. (L)inux, (M)ac OS, (W)indows

**Other suggestions?**
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Maths and engineering research: a clash of paradigms?

Assumptions

- (Applied, numerical) mathematics wants to advance in theory: from general to specific

- Practical engineering wants to build new machines and use modeling and simulation for component design and parameter selection: from specific to general

Result for me

Theoretical engineering with focus on control wants to apply the theory: general statements for a specific class of models

Questions

- What are the incentives to *bridge the communication gaps*?

- Are those gaps a result of *community paradigms*? (A math PhD student has to publish in math journals and an engineering PhD student in engineering journals)
Touched areas of applied mathematics

- **control systems** based on ordinary differential equations
- **differential-algebraic equations** (descriptor systems)
- graph theory
- switched systems
The standard feedback control loop

We have $r : \mathbb{I} \rightarrow \mathbb{R}$ as reference/setpoint, $e : \mathbb{I} \rightarrow \mathbb{R}$ as control error, $u : \mathbb{I} \rightarrow \mathbb{R}$ as input or control, $y : \mathbb{I} \rightarrow \mathbb{R}$ as output, $y_m : \mathbb{I} \rightarrow \mathbb{R}$ as measured output.

Figure: Standard control loop: single-input-single-output (SISO). Source: www.texample.net/tikz/examples/control-system-principles/.
Differential-algebraic equations (DAEs)

Definition

A set of equations of the form \(0 = F(t, x(t), \dot{x}(t))\), (short \(0 = F(t, x, \dot{x})\)) (1), with \(F: I \times D_x \times D_{\dot{x}} \rightarrow C^{n_e}\); \(D_x, D_{\dot{x}} \subseteq C^{n_s}\) are suitable open sets; \(n_e, n_s \in \mathbb{N}\); \(I \subseteq \mathbb{R}\) is a compact interval; is called a **set of differential-algebraic equations (DAE)**. Furthermore \(x: I \rightarrow C^{n_s}\) are called the state variables or unknown variables. \(t \in I\) is called the independent variable. If in addition to (1) an initial condition \(x(t_0) = x_0\) with \(t_0 \in I, x_0 \in C^{n_s}\) (2) exists, then (1), (2) is called **initial value problem (IVP)** and \(x_0\) is called the initial value. [Ste13]

If a set of equations can be written in the form

\[
\dot{x}_d(t) = \tilde{f}(t, x_d(t), x_a(t)) \\
0 = \tilde{g}(t, x_d(t), x_a(t))
\]

with \((\tilde{f}^T, \tilde{g}^T)^T: I \times \mathbb{R}^{n_d} \times \mathbb{R}^{n_a} \rightarrow \mathbb{R}^{n_e}; \quad n_d, n_a, n_e \in \mathbb{N}\); it is called a **semi-explicit DAE**. \(x_d\) are called differential state variables (or differential states) and \(x_a\) algebraic state variables (or algebraic states). In addition, we have \(n_d + n_a =: n_s\) as total number of states.
DAEs and the strangeness-index

**Hypothesis**

Consider $F \in C^\mu(D, \mathbb{R}^{n_{e}})$ with $D = I \times D_{x} \times D_{\dot{x}}$. Let there exist $\mu, d, a \in \mathbb{N}_0$, such that $\mathcal{L}_\mu \neq \emptyset$ and for every $z_0 \in \mathcal{L}_\mu$ there exist a sufficiently small neighborhood $B(z_0) \subset \mathcal{L}_\mu$, such that the following properties hold.

1. We have $\text{rank}(M_\mu(z_0)) = (\mu + 1)n_e - a$ on $\mathcal{L}_\mu$ and there exists a matrix function $Z_2$ with orthonormal columns and maximal rank $\text{rank}(Z_2) = a$ on $\mathcal{L}_\mu$, such that $Z_2^T M_\mu = 0$ on $\mathcal{L}_\mu$. Locally, we have that $Z_2 \in C^\mu(B(z_0), \mathbb{R}^{(\mu+1)n_e \times a})$.

2. We have $\text{rank}(Z_2^T \tilde{N}_\mu) = a$ on $\mathcal{L}_\mu$, where $\tilde{N}_\mu = N_\mu[l_{n_e} 0 \ldots 0]^T$, and there exists $T_1$ with orthonormal columns and maximal rank $\text{rank}(T_1) = d$, $d = n_e - a$, on $\mathcal{L}_\mu$, such that $Z_2^T \tilde{N}_\mu T_1 = 0$ on $\mathcal{L}_\mu$. Locally, we have that $T_1 \in C^\mu(B(z_0), \mathbb{R}^{n_e \times d})$.

3. We have $\text{rank}(F_\dot{x}(t, x, \dot{x})T_1(z_\mu)) = d$ on $\mathcal{L}_\mu$ and there exists $Z_1 \in \mathbb{R}^{n_{e} \times d}$ with orthonormal columns and maximal rank $\text{rank}(Z_1) = d$ on $\mathcal{L}_\mu$, such that $\text{rank}(Z_1^T F_\dot{x} T_1) = d$ on $\mathcal{L}_\mu$.

The smallest $\mu$ for which the hypothesis holds, is called the **s-index** (strangeness-index) of $F$. If $\mu = 0$, then $F(t, x, \dot{x}) = 0$ is called **s-free**.
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The swarm type low voltage DC microgrid model

Network as oriented undirected graph

Vertices $\rightarrow \text{(grid) nodes}$ where the SHSs are connected to the grid. Edges $\rightarrow \text{connections or power lines}$ between the SHSs. For the connection $jk$ of two grid nodes $j$ and $k$ with $j, k \in \{1, 2, ..., N\} =: \mathcal{N} \subset \mathbb{N}$, $j < k$ containing power line $h \in \{1, 2, ..., m\} =: \mathcal{M} \subset \mathbb{N}$, $m \in \{\eta \in \mathbb{N} \mid \eta \leq N(N-1)/2\}$, we define the edge-vertex incidence matrix

$$M := \begin{bmatrix}
m_{1,1} & \cdots & m_{1,N} \\
\vdots & \ddots & \vdots \\
m_{m,1} & \cdots & m_{m,N}
\end{bmatrix} \in \mathbb{R}^{m \times N},$$

with $m_{h,j} = 1$ and $m_{h,k} = -1$ and all others $m_{h,k} = 0$ with power line $h \in \mathcal{M}$ not connecting the SHS grid node $k \in \mathcal{N}$. We order the connections as $pq$, $pr$, $qr$ with $p, q, r \in \mathcal{N}$, $p < q < r \leq N$.

Remark: $M$ is singular!
Electrical equivalence circuit

**Figure:** Electrical equivalence circuit for two connected SHSs [Str15]
Modeling the SHS

We define

\[ f_j(t) = \begin{cases} g_{P,j}(t), & j \in \mathcal{N}_P \\ g_{C,j}(t), & j \in \mathcal{N}_C \end{cases} \]

with \( g_{P,j}: \mathbb{I} \to \mathbb{R}_{\geq 0} \) as producer current in ampere and \( g_{C,j}: \mathbb{I} \to \mathbb{R}_{< 0} \) as consumer current in ampere, \( j \in \mathcal{N} \).

The SHS as producer

For the SHS as producer, we get a controlled current source

\[ g_{P,\alpha}(t) = u_{\alpha}(t), \]

with \( u_{\alpha}: \mathbb{I} \to \mathbb{R} \) as input, \( \alpha \in \mathcal{N}_P \).

The SHS as consumer

- Constant power load (CPL): \( g_{C,\beta}(t) = \frac{P_{gh,\text{given},\beta}}{v_{gn,\beta}(t)} \) with \( \beta \in \mathcal{N}_C \)

- Constant current load (CCL): \( g_{C,\beta}(t) = i_{gh,C,\text{given},\beta} < 0 \), which has the (mathematical) advantage of being a linear equation.
Model equations: uncontrolled system

We reorder the equations and get the uncontrolled model for $t \in \mathbb{I}$.

**Uncontrolled model (\(*)**

\[
\begin{align*}
L_c \frac{d i_C(t)}{dt} & = v_C(t) - R_C i_C(t), \\
\hat{C}_{cap} \frac{d \hat{v}_{gn}(t)}{dt} & = \hat{i}_{cap}(t), \\
0 & = \hat{M}\hat{v}_{gn}(t) - v_C(t), \\
0 & = \hat{M}^T i_C(t) - \hat{i}_{gh}(t), \\
0 & = -\hat{i}_{cap}(t) - \hat{i}_{gh}(t) + \left[ \begin{array}{c} g_P(t) \\ g_C(t) \end{array} \right],
\end{align*}
\]
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Control: Droop control as standard control scheme

1. Control objectives
2. Droop control
Control objectives

1. **Bounded voltage deviation** in steady state from the reference voltage of the grid

   \[ |v_{gn,j}(t) - v_{gn,ref,j}| \leq e_v, \forall t \in I, j \in N, e_v \in \mathbb{R}_{>0} \]

   \[ \lim_{t \to \infty} |v_{gn,j}(t) - v_{gn,ref,j}| \leq e_v, \forall t \in I, j \in N. \]

2. **Power sharing** of the producing SHSs, i.e.,

   \[ \lim_{t \to \infty} P_{gh,\alpha}(t) = \lim_{t \to \infty} P_{gh,\tilde{\alpha}}(t) \forall \alpha, \tilde{\alpha} \in N_P. \]

**Next step:** Introduce droop control and check if the closed-loop model meets the control objectives. For which droop coefficients?
Droop control

Decentralized control scheme for the producing SHSs introducing a virtual resistance $d_{\text{droop,} \alpha} \in \mathbb{R}_{>0}$ in ohm, $\alpha \in \mathcal{N}_P$.

$$u_\alpha(t) = g_{P, \alpha}(t) = \frac{1}{d_{\text{droop,} \alpha}}(v_{\text{gn,ref}, \alpha} - v_{\text{gn,} \alpha}) = k_{\text{droop,} \alpha}(r_\alpha - y_\alpha)$$

Figure: Closed-loop model with droop control in standard feedback control loop representation without measurement dynamics or disturbances, $K_{\text{droop}} = \text{diag}(k_{\text{droop,} \alpha})$ and $v_{\text{gn,ref}} = \text{vector}(v_{\text{gn,ref,} j}, j \in \mathcal{N})$. 

Definition, modeling and control of swarm type direct current microgrids | Lia Strenge | September 1, 2016 | 28/40
Closed-loop model

Adding droop control and the nonlinear constant power load model

\[ g_{C,\beta}(t) = \frac{P_{gh,\text{given},\beta}}{v_{gn,\beta}(t)}, \beta \in \mathcal{N}_C \]

to the uncontrolled model \((\ast)\), we obtain the nonlinear DAE \((\ast\ast)\) for \(t \in \mathbb{I}\).

Closed-loop model \((\ast\ast)\)

\[
\begin{align*}
L_c \frac{di_c(t)}{dt} &= v_c(t) - R_c i_c(t), \\
\hat{C}_{cap} \frac{d\hat{v}_{gn}(t)}{dt} &= \hat{i}_{cap}(t), \\
0 &= \hat{M}\hat{v}_{gn}(t) - v_c(t), \\
0 &= \hat{M}^T i_c(t) - \hat{i}_{gh}(t), \\
0 &= -\hat{i}_{cap,P}(t) - \hat{i}_{gh,P}(t) + K_{\text{droop}}(v_{gn,\text{ref}} - \hat{v}_{gn,P}(t)), \\
0 &= -\hat{v}_{gn,C}(t) \circ \hat{i}_{cap,C}(t) - \hat{v}_{gn,C}(t) \circ \hat{i}_{gh,C}(t) + P_{gh,\text{given}}.
\end{align*}
\]
Closed-loop model

Adding droop control and the nonlinear constant power load model

\[ g_{C,\beta}(t) = \frac{P_{gh,\text{given},\beta}}{v_{gn,\beta}(t)}, \quad \beta \in \mathcal{N}_C \]

to the uncontrolled model (*), we obtain the nonlinear DAE (**) for \( t \in \mathbb{I} \).

Closed-loop model (**)

\[
\begin{align*}
L_c \frac{dx_{d,1}}{dt} &= x_{a,1} - R_c x_{d,1}, \\
\hat{C}_{cap} \frac{dx_{d,2}}{dt} &= x_{a,2}, \\
0 &= \hat{M} x_{d,2} - x_{a,1}, \\
0 &= \hat{M}^T x_{d,1} - x_{a,3}, \\
0 &= -x_{a,P,2} - x_{a,P,3} + K_{\text{droop}} v_{gn,\text{ref},P} - K_{\text{droop}} x_{d,P,2}, \\
0 &= -x_{d,C,2} \circ x_{a,C,2} - x_{d,C,2} \circ x_{a,C,3} + P_{gl,\text{given}}.
\end{align*}
\]
**Lemma**

The closed-loop model (**) is strangeness-free with $d = m + N$, $a = m + 2N$ if $v_{g_n, \beta}(t) > 0$, $\beta \in \mathcal{N}_C$, $\forall t \in \mathbb{I}$.

**Proof.**

We omit the time argument of the states hereafter and write (**) in the form $\dot{x}_d = \tilde{f}(x_d, x_a)$, $0 = \tilde{g}(x_d, x_a)$ with $x_d := (i_T^T, \hat{v}_{g_n}^T)^T$, $x_a := (v_c^T, \hat{i}_{cap}^T, \hat{i}_{gh}^T)^T$ and $(\tilde{f}^T, \tilde{g}^T)^T$ as right side of (**). We need to show that $\tilde{g}_{x_a}$ is regular.
Proof

Proof. 

$$\tilde{g}_{xa} = \begin{bmatrix} -I_m & O_{m \times N} & O_{m \times N} \\ O_{p \times m} & -I_p & O_{p \times l} \\ O_{l \times m} & -\text{diag}(\hat{v}_{gn,C}) & -\text{diag}(\hat{v}_{gn,C}) \end{bmatrix} \begin{bmatrix} -I_p & O_{p \times l} \\ O_{l \times p} & -\text{diag}(\hat{v}_{gn,C}) \end{bmatrix}$$

Let $\tilde{P}$ be a suitable permutation matrix of dimension $m + 2N$. 
Proof (II)

Proof.

We get that

\[
\tilde{P}\tilde{g}_x = \begin{bmatrix}
-\mathcal{I}_m & \mathcal{O}_{m\times N} \\
\mathcal{O}_{p\times m} & -\mathcal{I}_p \\
\mathcal{O}_{l\times m} & -\mathcal{I}_l \\
\mathcal{O}_{m\times N} & -\text{diag}(\hat{V}_{gn,c}) \\
\mathcal{O}_{l\times p} & -\text{diag}(\hat{V}_{gn,c}) \\
\mathcal{O}_{N\times N} & -\mathcal{I}_N
\end{bmatrix}.
\]

\(\tilde{P}\tilde{g}_x\) is regular since it is an upper triangular matrix with full diagonal for \(\text{diag}(\hat{V}_{gn,c}) > 0\forall t \in \mathbb{T}\).

Hence, the DAE (**) is strangeness-free.
Technical interpretation and conclusion

Preliminary results for droop control (based on exemplary simulation)

- Constant offset of maximum 2.4 volt, i.e., 5 percent
- Equal power sharing achieved for simple network topologies

Conclusion
We have a modular mathematical representation of the swarm type low voltage DC microgrid allowing for control design and simulation

Open issues until a possible contribution to the technology dev.

- Mode switching between producer and consumer (prosumer)
- Inclusion of $P_{gh,\text{max}}$ in order to evaluate proportional power sharing
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Work in progress

- Simulation study on parameter variations for linear and nonlinear model and a fixed network topology
- Test bed development to verify modeling assumptions by experimental data
Switching: mode changes in the SHS (I - time-based)

We have a partition of the time interval \( I = \bigcup_{\nu=0}^{\mu} I_\nu \), with \( I_\nu = [t_\nu, t_{\nu+1}) \), \( I_\mu = [t_\mu, \infty) \) and \( T := \{t_0, t_1, t_2, \ldots, t_\nu, \ldots, t_\mu\} \).

Assumption

We assume a fixed \( q(0) \in \{0, 1\}^N \) for all \( t \in I_0 \). At \( t = t_1 \), the first power flow change including a mode change from producer to consumer or vice versa of at least one SHS \( j \in \mathcal{N} \) takes place with a sign change of the corresponding \( i_{gh,j} \) and therefore the switching of the corresponding \( q(0),j \). Hence, we have a new \( q(1) \in \{0, 1\}^N \) for all \( t \in I_1 \). Analogously, at \( t = t_\nu \in T \), the \( \nu \)-th mode power flow change including a sign change of at least one \( i_{gh,j}, j \in \mathcal{N} \) takes place and the corresponding \( q(\nu-1),j \) switch(es). We have \( q(\nu) \in \{0, 1\}^N \) for all \( t \in I_\nu \). Hence, we have \( q: T \rightarrow \{0, 1\}^N; q = q(t_\nu) = q(\nu) \).

Hence,

\[
f_j(t, q_j): = \begin{cases} g_{P,j}(t) & , \quad q_j = 1 \\ g_{C,j}(t) & , \quad q_j = 0 \end{cases},
\]

with \( g_{P,j}: I \rightarrow \mathbb{R}_{\geq 0} \) as producer current in ampere and \( g_{C,j}: I \rightarrow \mathbb{R}_{< 0} \) as consumer current in ampere, \( j \in \mathcal{N} \).
Future work

- Analytic derivation of boundaries for stable operation of the swarm type DC microgrid
- Measurements to verify modeling assumptions by experimental data
Bibliography I

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Differential-Algebraic Equations.  

Lia Strenge.  
Modeling and simulation of a droop controlled swarm type low voltage DC microgrid in a DAE framework.  
Summary and discussion

Summary

- Strangeness-free closed-loop model without switching
- Bi-directional power flow by discrete event systems or hybrid system theory

Thank you for your attention!
Advantages of strangeness-free DAEs

- Closer to physical intuition compared to a manually derived ordinary differential equation (ODE) in fewer variables
- A large part of the ODE theory has been transferred to strangeness-free DAEs
- Decreased numerical differentiation and hence increased numerical stability compared to problems of higher s-index
- No hidden constraints
Research projects at Control Systems Group

Biomedical Engineering
- Inertial Sensor-Based Gait Analysis
- Controlled Functional Electric Stimulation for Rehabilitation Purposes

Chemical Process Engineering
- Hyperbolicity of Quasi-Linear PDEs for General Isotherms
- Crystal Shape Control
- Control of Preferential Crystallization Processes

Power Systems
- Hybrid Control Systems
- Control of Discrete Event Systems

HTS-Systems
- Mathematical Modelling of Viral Evolution and Cultivation Processes
- Bioprocess Engineering

Control Concepts for Microgrids

APPLICATION

THEORY

Hierarchical and Cooperative Control