Abstracts

Ilka Agricola

1. The classification of naturally reductive homogeneous spaces
(based on joint work with Ana Cristina Ferreira and Thomas Friedrich)

Abstract:
We present a new method for classifying naturally reductive homogeneous spaces - i.e. homogeneous Riemannian manifolds admitting a metric connection with skew torsion that has parallel torsion and curvature. This method is based on a deeper understanding of the holonomy algebra of connections with parallel skew torsion on Riemannian manifolds and the interplay of such a connection with the geometric structure on the given Riemannian manifold. It allows to reproduce by easier arguments the known classifications in dimensions 3, 4, and 5, and yields as a new result the classification in dimension 6.

2. G-structures and their remarkable spinor fields
(based on joint work with Julia Becker-Bender, Simon Chiossi, Thomas Friedrich, Jos Höll, and Hwajeong Kim)

Abstract:
In this talk, I will describe how several interesting G-structures on Riemannian manifolds (in particular almost contact structures, 3-contact structures, SU(3)-structures, and G_2-structures) can be described in a systematic way through remarkable spinor fields and field equations that they satisfy - generalizing the well-known equations for Riemannian Killing spinors, generalized Killing spinors, Killing spinors with torsion, and quasi-Sasakian Killing spinors. Applications include eigenvalue estimates for Dirac operators with skew torsion, cone constructions for G-manifolds admitting a characteristic connection, and embedding theorems for hypersurfaces.

Sergey Cherkis

Yang-Mills Instantons and Hypercomplex Connections

Abstract:
The first lecture will be focussed on Yang-Mills Instantons and Self-dual Gravitational Instantons, introducing quivers and bows, and using them to construct hypercomplex Yang-Mills connections on bow moduli spaces. We shall discuss the geometry of the moduli spaces of such hypercomplex connections.

In the second lecture we formulate the inverse Nahm transform for Yang-Mills instantons on ALF spaces. Here the relevant bow data will emerge as an index bundle of the family of Dirac operators in the instanton background. This is the work in progress with Mark Stern and Andres Larrain-Hubach. Time permitting, we shall also discuss the relevant string theory brane picture and noncommutative deformations of the bow moduli spaces.

Marco Gualtieri

Log symplectic vs. generalized complex structures

Abstract:
In the first talk, I will review various special Lie algebroids and Courant algebroids related to the logarithmic tangent bundle and explain how these inform and simplify the study of geometric structures with singularities along divisors.

In the second talk I will explain how these are related to classes of generalized complex structures which are generically symplectic.

Martin Guest

Classification of 2-dimensional N = 2 supersymmetric theories, revisited

Abstract:
Cecotti and Vafa (Commun. Math.Phys. 1993) proposed a classification program for N = 2 supersymmetric conformal theories and their massive deformations, in dimension 2. Such deformations correspond to very special (conjectural) solutions of the tt* (topological-antitopological fusion) equations, a nonlinear system of pde. They are constructed by combining holomorphic data with antiholomorphic data, in the spirit of twistor theory. In the context of mirror symmetry, the holomorphic objects associated to these conjectural solutions have rich geometric interpretations as unfoldings of singularities or quantum cohomology of certain Kaehler manifolds.

Dubrovin observed that the tt* equations have a zero curvature formulation - they are a special case of the (pluri)harmonic map equations, familiar from differential geometry. Moreover they are a special case of the harmonic bundle equations, or Hitchin equations - they can be regarded as equations for a Hermitian metric on a certain moduli space. In the local theory of harmonic maps the "holomorphic-harmonic correspondence" is well known as the DPW representation or generalized Weierstrass representation. In certain situations, the Hitchin-Kobayashi correspondence goes deeper and gives global results. These "integrable systems aspects" of the tt* equations were not explored 20 years ago, but are again potentially of interest because of recent work on tt* geometry in 3 and 4 dimensions by Cecotti, Gaiotto, and Vafa.

In these lectures we shall go back to 2 dimensions and explain an example where the tt* equations can be solved rigorously and explicitly by using the full power of integrable systems methods. This example is related to the well-known Toda equations, and hence to several other parts of mathematics. In this case the original proposal of Cecotti and Vafa can be carried out completely, confirming all their expectations.

We take the viewpoint of differential geometry. In the first lecture we shall give some background and explain the results with a minimum of technicalities. In the second lecture we shall describe the methods of solving the equations. These methods are all well known in their own context, but together they represent a broad accumulation of ideas. Whether they can be implemented for other 2-dimensional examples (or even in dimensions 3 and 4) is not clear at the present time. However, they do provide another case where deep predictions from quantum field theory can be justified mathematically.

Nigel Hitchin

The Higgs bundle moduli space

Higgs bundles and diffeomorphism groups

Abstract:
Replacing a compact group by the group of symplectic diffeomorphisms of a manifold has been used many times to motivate geometric theorems. In this talk we take the Higgs bundle equations over a compact Riemann surface with group the symplectic diffeomorphisms of a 2-sphere and link it to the study of folded hyperkahler 4-manifolds. We conjecture the existence of an infinite dimensional analogue of Teichmuller space which parametrizes path geometries on the surface.

Chris Hull

Geometry, Non-Geometry and String Theory

Abstract:
One of the ways in which string theory differs from conventional field theories is that is has duality symmetries. Duality symmetries, such as T-duality, will be introduced and some of the implications discussed. Their role in the construction of so-called non-geometric backgrounds in string theory will be reviewed, such as T-folds which have T-duality transition functions. Some of the mathematical framework that is useful in discussing such matters
will be developed.

Double Geometry, Generalised Geometry and String Theory

Abstract:
String theory in a background with a toroidal fibration is naturally formulated in a double geometry, with non-trivial dynamics in the full doubled space-time. This geometry will be developed and applied to string theory, and its relation to generalised geometry will be discussed.

Spiro Karigiannis

Talk 1: An introduction to G_2 geometry

Abstract:
I will give an introduction to the Riemannian geometry and the topology of G_2 manifolds. I will stress the similarities to and the differences from Kähler and Calabi-Yau geometry. Topics to be discussed include: examples; calibrated submanifolds; and moduli spaces.
This talk is suitable for graduate students familiar with Riemannian geometry and differential forms.

Talk 2: G_2 conifolds: A survey

Abstract:
I will begin with an introduction to G_2 conifolds. I will then proceed to give a detailed survey of recent mathematical developments on G_2 conifolds, including desingularization, deformation theory, and possible constructions of G_2 conifolds. This includes separate joint works of myself with Jason Lotay and with Dominic Joyce.

Andre Neves

Min-max Theory in Geometry

Abstract:
Min-max methods in Geometry were first used around 100 years ago to show that every metric on a 2-sphere has a closed geodesic. Since then many other applications have been found. I will talk about those and then focus on my joint work with Fernando Marques where we have found new applications for the method within the last years.

Boris Pioline

BPS indices and quaternionic geometry

Abstract:
In quantum field theories or string theory vacua with extended supersymmetry, BPS states play a central role as they can be reliably followed from weak to strong coupling. BPS indices count (signed) BPS bound states, and are locally constant away from walls of marginal stability. In the context of four-dimensional gauge theories with N=2 supersymmetry, I will explain how the BPS indices combine into a hyperkahler metric and hyperholomorphic connection on a natural moduli space, both of which being smooth across the walls provided the BPS indices jump according to the Joyce-Song-Kontsevich-Soibelman wall-crossing formula. If time permits, I will also discuss the extension of these constructions to quaternion-kahler geometry, which is relevant for string theory vacua with N=2 supersymmetry.

James Sparks

Exact results in AdS/CFT from localization

Abstract:
It has recently been understood how to define supersymmetric quantum field theories on rather general classes of compact Riemannian manifolds, equipped with certain extra geometric structures. An important feature of such constructions is that the path integral localizes onto supersymmetric field configurations (a sort of infinite-dimensional version of the Duistermaat-Heckman theorem). Under favourable conditions one can calculate a variety of observables exactly, in closed form. I will describe some of this progress, and explain how it may be used to obtain exact results in the AdS/CFT correspondence. Geometrically this involves a filling problem where one solves the Einstein equations on some manifold M_{d+1}, with conformal boundary M_d on which the conformal field theory is defined.

Maxim Zabzine

5D Yang-Mills, contact geometry and special functions

Abstract:
I will review the use of localization techniques for 5D supersymmetric Yang-Mills theory. The role of contact geometry will be stressed. The explicit results for the case of the toric Sasaki-Einstein manifolds will be discussed and new special functions responsible for the different counts will be presented.