Titles and abstracts

Giovanni Bellettini

New results on the area of the graph of nonsmooth maps from the plane to the plane

In minimal surface theory, for instance in the cartesian Plateau problem, it turns out to be useful to extend, via a relaxation procedure, the concept of area to the graph of a nonsmooth scalar function. In this one-codimensional case, the extension is well understood, it is naturally defined on functions with bounded variation and admits there an integral representation. A similar problem for codimension two minimal surfaces appears to be, at the present moment, largely open. We shall discuss some recent results concerning the expression of the relaxed area functional for suitable nonsmooth maps from the plane to the plane; in particular, we shall focus attection to piecewise constant maps taking three noncollinear values, and jumping on a tripod, and to maps which are smooth out of suitable jump curve.

Reto Buzano

The moduli spaces of two-convex embedded spheres and tori

It is interesting to study the topology of the space of smoothly embedded n-spheres in R^{n+1}. By Smale's theorem, this space is contractible for $n = 1$ and by Hatcher's proof of the Smale conjecture, it is also contractible for $n = 2$. These results are of great importance, generalising in particular the Schoenflies theorem and Cerf's theorem. In this talk, I will explain how mean curvature flow with surgery can be used to study a higher-dimensional variant of these results, proving in particular that the space of two-convex embedded spheres is path-connected in every dimension n. We then also look at the space of two-convex embedded tori where the question is more intriguing and the result in particular depends on the dimension n. This is all joint work with Robert Haslhofer and Or Hershkovits.

Gilles Carron

Scalar curvature and strong A_∞ weight

Abstract: This is a joint work with Clara Aldana (Luxembourg) and Samuel Tapie (Nantes) . I will explain some relationship between critical integral finiteness for the scalar curvature and the notion of Strong A_∞ weight introduced by G. David and S. Semmes. Applications for conformal metric on the Euclidean space will be given, these results will be a generalisation of earlier works of M. Bonk, J. Heinonen, E. Saksman and of recent results of Y. Wang.

Alix Deruelle

A relative entropy for expanders of the Harmonic map flow

We focus on the uniqueness question for (expanding) solutions of the Harmonic map flow coming out of smooth 0-homogeneous maps with values into a closed Riemannian manifold. We introduce a relative entropy to prove the existence of two expanding solutions associated to any suitable solution coming out of a 0-homogeneous map by a blow-up and a blow-down process. Generic uniqueness of expanding solutions coming out of the same 0-homogeneous map of zero relative entropy is investigated.

Baptiste Devyver

Gradient estimates for the heat kernel on non-compact manifolds

Euclidean-type estimates for the heat kernel on a non-compact Riemannian manifold are well-understood: thanks to the works of many authors (Varopoulos, Saloff-Coste, Grigoryan, Coulhon, etc), they are known to be characterised by the validity of certain functional inequalities, such as the Sobolev inequality for instance.
Much less is known concerning the behaviour of the gradient of the heat kernel. By a famous result of Li and Yau, pointwise bounds of Euclidean-type hold if the Ricci curvature is non-negative, but no optimal result is known beyond this quite stringent curvature assumption.
We will present some new recent results in this respect, for manifolds having a "small amount" of negative Ricci curvature at infinity, in an integral sense. In particular, we will discuss gradient estimates for the heat kernel on asymptotically locally Euclidean manifolds. Our results also have applications concerning the heat equation associated with vector-valued operators, such that the Hodge Laplacian on differential forms.

Panagiotis Gianniotis

The bounded diameter conjecture for 2-convex mean curvature flow

In this talk I address the bounded diameter conjecture for the mean curvature flow of smooth 2-convex hypersurfaces in R^{n+1}. In joint work with Robert Haslhofer, we prove that the intrinsic diameter of the evolving hypersurfaces is controlled, up to the first singular time, in terms of geometric information of the initial hypersurface. Moreover, this diameter estimate leads to sharp L^{n-1} estimates for the curvature at each time. Our estimates extend to mean curvature flow with surgery, which allows us to obtain the optimal L^{n-1} estimate for any level set flow starting from a smooth 2-convex hypersurface. This improves the $L^{n-1-\varepsilon}$ curvature estimate that was previously established in work of Head and Cheeger-Haslhofer-Naber.

Niels Martin Møller

Self-Translating Solitons for the Mean Curvature Flow

I will discuss a couple of new results on the classification problem for complete self-translating hypersurfaces for the codimension one mean curvature flow. Such surfaces show up as singularity models in the flow (along with other types of solitons, e.g. self-shrinkers), and they are minimal surfaces in $\mathbb{R}^{n+1}$ endowed with certain conformally changed (via an exponential factor) Riemannian background metric. This is joint work with Francesco Chini (U Copenhagen).

Ovidiu Munteanu

Structure of Ricci solitons

We will cover some recent results about the geometry and topology of Ricci solitons. These are self similar solutions of the Ricci flow that are important in understanding its singularities. Several classification results of Ricci solitons are known in low dimensions, and recent efforts have focused on their structure in dimension four. We will overview recent development about the asymptotic geometry of four dimensional shrinking solitons.

Michele Rimoldi

The Frankel property for self-shrinkers from the viewpoint of elliptic PDE's

In many instances, properly immersed self-shrinkers behave like closed minimal hypersurfaces of the standard sphere. In this latter setting, it is well known that any two closed minimal immersed hypersurfaces must intersect. Actually the ambient space can be generalized to a compact Riemannian manifold with strictly positive Ricci curvature. This is called the Frankel property after the celebrated paper by T. Frankel. Starting from the work by G. Wei and W. Wylie, where the case of compact hypersurfaces is considered, we investigate the validity of the (smooth) properly embedded Frankel property for self-shrinkers of the MCF. In particular, we prove that two properly embedded self-shrinkers in Euclidean space that are sufficiently separated at infinity must intersect at a finite point. The proof is based on a localized version of the Reilly formula applied to a suitable f-harmonic function with controlled gradient. Moreover, in the immersed case, we will show how tools from potential theory of weighted manifolds permit to prove half-space type properties for self-shrinkers. This is a joint work with D. Impera and S. Pigola.

Felix Schulze

Optimal isoperimetric inequalities for 2-dimensional surfaces in Hadamard-Cartan manifolds in any codimension

Let (M^n, g) be simply connected, complete, with non-positive sectional curvatures, and Σ a 2-dimensional closed integral current (or flat chain mod 2) with compact support in M. Let S be an area minimising integral 3-current (resp. flat chain mod 2) such that $\partial S = \Sigma$. We use a weak mean curvature flow, obtained via elliptic regularisation, starting from Σ, to show that S satisfies the optimal Euclidean isoperimetric inequality: $6\sqrt{\pi}\,\mathbf{M}[S] \leq (\mathbf{M}[\Sigma])^{3/2}$. We also obtain an optimal estimate in case the sectional curvatures of M are bounded from above by $-\kappa < 0$ and characterise the case of equality. The proof follows from an almost monotonicity of a suitable isoperimetric difference along the approximating flows in one dimension higher and an optimal estimate for the Willmore energy of a 2-dimensional integral varifold with first variation summable in L^2.

Peter Topping

Pyramid Ricci flows

I will discuss some recent theory for starting Ricci flows in situations where no classical solution is expected to exist. My plan is to emphasise the (geometric) PDE theory rather than the applications, with the so-called Pyramid Extension Lemma a key target. Joint work with Miles Simon and Andrew McLeod.

Burkhard Wilking

Ricci flow of manifolds with almost nonnegative complex sectional curvature

If a curvature condition is preserved by the Ricci flow and one starts with a initial metric that almost satisfies the condition, e.g. complex sectional curvature is bounded below by a small negtive constant, then its well known that for some time the complex curvature stays bounded below by a small constant. However, the classic approach only allows to estiamte the time interval in terms of an upper curvature bound of the initial metric. We explain that this upper curvature bounds can be replaced with a lower bound on volume of unit balls. We present various applications and generalizations.