

Lecture 1

Topological Recursion
B.EYNARD, IPHT Saclay, CRM

Outline

1. Intro
 - generalities
 - example
 - computation
 - Generalization
2. Topological Recursion
 - Def (s)
3. Applications
 - Enum geom
 - Topological Vertex
 - Knots

Lecture 2

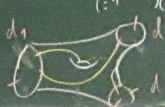
Some Examples

- 1) Kontsevich-Witten

$$\begin{cases} x(z) = z^2 \\ y(z) = z \\ B(z_1, z_2) = \frac{dz_1 dz_2}{(z_1 - z_2)^2} \end{cases} \rightarrow w_{g,n}(z_1, \dots, z_n) = 2^{2g+n} \sum_{d_1, \dots, d_n} \prod_{i=1}^n \frac{(2d_i+1)!! dz_i}{z_i^{2d_i+2}} \langle \tau_{d_1} \dots \tau_{d_n} \rangle_g$$
- 2) Weil-Petersson - Mirzakhani

$$\begin{cases} x = z^2 \\ y = \frac{1}{4\pi} \sin 2\pi z \end{cases} \quad w_{g,n} = \sum_{d_1, \dots, d_n} \prod_{i=1}^n \frac{(2d_i+1)!! dz_i}{z_i^{2d_i+2}} \langle e^{2\pi^2 K_1} \tau_{d_1} \dots \tau_{d_n} \rangle_g$$

$$= \int_{\mathbb{R}^n} \prod_{i=1}^n L_i dL_i e^{-\sum_{i=1}^n L_i^2 \psi_i} \langle e^{2\pi^2 K_1 + \frac{1}{2} \sum_{i=1}^n L_i^2 \psi_i} \rangle_g$$
- 3) Catalan

$$\begin{cases} x = z + \frac{1}{z} \\ y = \frac{1}{z} \end{cases} \quad w_{g,n} = \sum_{d_1, \dots, d_n} C_{g,n}(d_1, \dots, d_n) \prod_{i=1}^n \frac{dx_i}{x_i^{d_i+1}} \quad x_i = x(z_i)$$

- 4) Topological Vertex

$$\begin{aligned} x &= -\log(1-z) - f \log z \\ y &= -\log z \\ e^{-x-fy} + e^{-y} &= 1 \quad \text{Mirror of } \mathbb{CP}^3 \end{aligned}$$

$$w_{g,n} = f^{-n} (f(f+1))^{g-1} \sum_{\mu_1, \dots, \mu_n} \prod_{i=1}^n \frac{\Gamma(\mu_i(f+1))}{\mu_i! \Gamma(\mu_i f)} \mu_i e^{-\mu_i x_i} \left\langle \prod_{i=1}^n \frac{1}{1-\mu_i \psi_i} \Lambda(f) \Lambda(f) \Lambda(-1-f) \right\rangle_g$$

$$= \sum_{\mu_1, \dots, \mu_n} \prod_{i=1}^n (\mu_i e^{-\mu_i x_i} d\mu_i) \quad \text{GW}_{g,n}(\mathbb{CP}^3, \mu_1, \dots, \mu_n)$$
- 5) Toric - CY3 fold X, brane Z

$$P(e^{-x}, e^{-y}) = 0 \quad \text{Mirror}$$

$$w_{g,n} = \sum_{\beta \in H_2(X)} \sum_{\mu_1, \dots, \mu_n} \prod_{i=1}^n \frac{b_i(x) - t_i \beta_i}{\mu_i!} \mu_i e^{-\mu_i x_i} \quad \text{GW}_{g,n}(X, Z, \beta, \mu_1, \dots, \mu_n)$$

Lecture 3

Plan

- Deformations - Special geometry - Form-cycle duality
- Quantum Curves
- Hirota, Integrability
- CFT

$$e^{-x} e^{fy} + e^{-y} = 1$$

$$\mathcal{Y} \begin{cases} x = -\log(1-z) - f \log z \\ y = -\log z \\ B(z_1, z_2) = \frac{dz_1 dz_2}{(z_1 - z_2)^2} \end{cases}$$

Thm

$$\omega_{g,n} = 2^{3g-3+n} \sum_{d_1 \dots d_n} \prod_{i=1}^n d\xi_{d_i}(z_i) \int_{\mathcal{M}_{g,n}} \Lambda_{\mathcal{Y}} \prod_{i=1}^n \psi_i^{d_i}$$

$$d\xi_d(z) = \operatorname{Res}_{z \rightarrow a} B(z, z') (x(z') - x(a))^{-\frac{d}{2}} \times \left(\frac{-(z-d-1)!!}{z^d} \right)$$

$$\Lambda_{\mathcal{Y}} = e^{\left(\sum_k \hat{t}_k \kappa_k + \frac{1}{2} \sum_{\delta=(r,r')} \sum_{k,l} B_{k,l} \psi_r^k \psi_{r'}^l \right)}$$

$$\sum_k \hat{t}_k u^{-k} = -\log \left(\frac{u\sqrt{u}}{2\sqrt{\pi}} \int y dx e^{-u(x-x(a))} \right)$$

$$\sum_{k,l} B_{k,l} u^{-k} v^{-l} = \frac{\sqrt{uv}}{2\pi} \int B(z, z') e^{-u(x(z)-x(a))} e^{-v(x(z')-x(a))} dz$$