Geometric Flows and the Geometry of Space-time
September 19-23, 2016, Hamburg

Abstracts

Lars Andersson:

Spin geometry of the Kerr spacetime

I will introduce the 2-spinor calculus and show how the geometry of the Kerr spacetime is encoded in terms of the Killing spinor and its derivatives. I will explain how these facts enable one to carry out efficient symbolic calculations for fields on the Kerr spacetime and indicate some applications.

Potentials, symmetries and conservation laws

Massless fields on the Kerr spacetime are governed by the Teukolsky Master Equation, and the Teukolsky-Starobinsky Identities. Solutions can be represented in terms of Deby potentials. In this talk I will show how these facts lead to insights on symmetry operators and conservation laws for fields on the Kerr spacetime, including linearized gravity.

Helga Baum:

Lorentzian manifolds - holonomy and spinors

- Basic notions (metric, type of vectors and curves, curvature)
- Holonomy groups (short explantion of the classification result)
- Lorentzian spin geometry (spin structures, parallel spinors and consequences for curvature and holonomy)
- Space-like submanifolds (induced curvature and induced spinor field equations)
- Globally hyperbolic manifolds (notion, smooth splitting result, well-poseness of the Cauchy problem for normally hyperbolic operators)

Spiros Cotsakis:

Global hyperbolicity and the completeness of geometric flows

We discuss consequences for Lorentzian geometry of the fact that the generic form of the metric in various geometric flows, such as the Ricci flow and the Horava-Lifshitz flow, is only invariant under the foliation-preserving diffeomorphisms, not the full group of space-time transformations. These are related to obvious difficulties with the formulation of standard causality theory, space-time geodesic completeness, and therefore obstructions to connecting the singularity models of geometric flows to the usual singularity theorems of general relativity. We review results on sufficient conditions of geodesic completeness (associated primarily with the work of Y. Choquet-Bruhat), and indicate how one may talk intelligently about curves of infinite space-time length in the context of geometric flows. We also review recent joint work with Ioannis Bakas on the renormalization of the space-time length and its relation to the Perelman (reduced) length in the special case of curves which have the property of being uniformly timelike. We also briefly discuss the more involved issue of proving geodesic completeness in the general case.

Pau Figueiras:

Numerical Ricci flows and black holes

In this talk I will describe a novel use of Ricci flows that has attracted a lot of interest in recent years in the theoretical physics community. Black hole spacetimes are (Lorentzian) Einstein manifolds that play a central role in our understanding of general relativity, Einstein's theory of gravity. In this talk I will explain how one can use Ricci flows to find, numerically, equilibrium black hole spacetimes and I will provide some simple examples, emphasising their physical relevance. Note that the flows that often arise in black hole physics are Ricci flows on Lorentzian non-compact manifolds, with various asymptotic boundary conditions, including (but not restricted to) asymptotic flatness.

Gary Gibbons:

Black Holes

This will be an introduction to the Theory of Black Holes which will cover
1) The Oppenheimer Snyder Gravitational account of spherically Symmetric gravitational collapse and the Krusakl Manifold
2) Rotating black holes: Kerr solution
3) Uniqueness Theorems for Stationary and Static Black holes
4) Hawking Evaporation.
I will assume a first course in General Relativity.

Mark Haskins:

Tba

Jason Lotay:

Laplacian flow in G2 geometry

A key challenge in Riemannian geometry is to find Ricci-flat metrics on compact manifolds. All non-trivial examples of such metrics have special holonomy, and the only special holonomy metrics which can occur in odd dimensions must be in dimension 7 and have holonomy G_2. I will describe a proposed geometric flow method for finding metrics with holonomy G_2, called the Laplacian flow, and recent progress on this flow. This is joint work with Yong Wei (University College London).

Jan Metzger

Mass, area and volume

In initial data sets of General Relativity the global physical quantities like mass and center of mass play an important role. In this course we consider the relation between the mass and the isoperimetric profile of an asymptotically flat manifold.

Thomas Leistner:

Cauchy problems for Lorentzian manifolds with special holonomy

Lorentzian manifolds with parallel null spinor or, more generally, parallel null vector arise naturally in general relativity, as supersymmetric supergravity backgrounds, but also in the theory of Lorentzian manifolds with special holonomy. In analogy to the Cauchy problem in general relativity, we study the corresponding Cauchy problems for these manifolds: Can a given Riemannian manifold (M,g) be embedded (as a Cauchy hypersurface) in a Lorentzian manifold with parallel null vector/ spinor field? We derive the constraint and the evolution equations for this problem. By reducing them to a system in Cauchy-Kowalevski form and moreover to a quasilinear symmetric hyperbolic system, we show that the evolution equations have a unique (analytic/smooth) solution provided the initial data are analytic/smooth and satisfy the constraints. Moreover, for Riemannian manifolds obeying the constraint conditions, we derive a local normal form and use the classification of Lorentzian holonomy groups to describe their special geometry. As an application of our results to Riemannian geometry we obtain a classification of the local geometry of Riemannian manifolds with generalised imaginary Killing spinors.

This is joint work with H. Baum and A. Lischewski (both Humboldt University Berlin).

Oliver Schnürer:

Geometric flow equations

In this minicourse, we study hypersurfaces that solve geometric evolution equations. More precisely, we investigate hypersurfaces that evolve with a normal velocity depending on a curvature function like the mean curvature or Gauß curvature. In three lectures, we will address

- Hypersurfaces, principal curvatures and evolution equations for geometric quantities like the metric and second fundamental form.
- The convergence of convex hypersurfaces to round points. Here, we will also show some computer algebra calculations.
- The evolution of graphical hypersurfaces under mean curvature flow.