Geometry and classification of string AdS backgrounds

George Papadopoulos

King's College London

Field equations on Lorentzian space-times

University of Hamburg
Germany
19-23 March 2018

Work presented is in collaboration with

Sam Beck, Ulf Gran, Jan Gutowski, Sebastian Lautz, Alexander Haupt

Supersymmetry of AdS backgrounds

- The classification of AdS supergravity backgrounds, $AdS_n \times_w M^{D-n}$, is a longstanding problem raised in the context of supergravity compactifications [Freund-Rubin] that goes back into the early '80s
- Recently they have found applications in string theory and in M-theory as near horizon geometries for black holes and branes
- In AdS/CFT, supergravity $AdS_n \times_w M^{D-n}$ solutions are associated to the vacuum state of a dual superconformal theory. Fluctuations of $AdS_n \times_w M^{D-n}$ are associated to certain gauge invariant operators of the dual theory.

Supersymmetry of AdS backgrounds

- The classification of AdS supergravity backgrounds, $AdS_n \times_w M^{D-n}$, is a longstanding problem raised in the context of supergravity compactifications [Freund-Rubin] that goes back into the early '80s
- Recently they have found applications in string theory and in M-theory as near horizon geometries for black holes and branes
- In AdS/CFT, supergravity $AdS_n \times_w M^{D-n}$ solutions are associated to the vacuum state of a dual superconformal theory. Fluctuations of $AdS_n \times_w M^{D-n}$ are associated to certain gauge invariant operators of the dual theory.

Supersymmetry of AdS backgrounds

- The classification of AdS supergravity backgrounds, $\mathrm{AdS}_n \times_w M^{D-n}$, is a longstanding problem raised in the context of supergravity compactifications [Freund-Rubin] that goes back into the early '80s
- Recently they have found applications in string theory and in M-theory as near horizon geometries for black holes and branes
- In AdS/CFT, supergravity $\mathrm{AdS}_n \times_w M^{D-n}$ solutions are associated to the vacuum state of a dual superconformal theory. Fluctuations of $\mathrm{AdS}_n \times_w M^{D-n}$ are associated to certain gauge invariant operators of the dual theory.

Objectives

- Describe some aspects of the geometry of the internal space M^{D-n} in 10- and 11-dimensional supergravities. These include new Lichnerowicz type of theorems
- Present the classification of warped AdS backgrounds, $\mathrm{AdS}_n \times_w M^{D-n}$, with the most general allowed fluxes in $D = 10$ and 11 dimensions that preserve more than 16, $N > 16$, supersymmetries

Objectives

- Describe some aspects of the geometry of the internal space M^{D-n} in 10- and 11-dimensional supergravities. These include new Lichnerowicz type of theorems
- Present the classification of warped AdS backgrounds, $\mathrm{AdS}_n \times_w M^{D-n}$, with the most general allowed fluxes in $D = 10$ and 11 dimensions that preserve more than 16, $N > 16$, supersymmetries

New developments

The problem has become tractable because of three key recent developments

- The Killing spinor equations (KSEs) of type II 10- and 11-dimensional supergravities have been solved over the AdS subspace for all $AdS_n \times_w M^{D-n}$ backgrounds with the most general allowed fluxes leading to the identification of the number of supersymmetries that can be preserved [Beck, Gutowski, GP]
- The proof of the homogeneity theorem which states that all backgrounds which preserve more than $1/2$ of supersymmetry are homogeneous Lorentzian spaces [Figueroa-O'Farrill, Hustler]
- The identification of all Killing superalgebras of AdS backgrounds in 10- and 11-dimensions [Beck, Gutowski, Gran, GP]

New developments

The problem has become tractable because of three key recent developments

- The Killing spinor equations (KSEs) of type II 10- and 11-dimensional supergravities have been solved over the AdS subspace for all $AdS_n \times_w M^{D-n}$ backgrounds with the most general allowed fluxes leading to the identification of the number of supersymmetries that can be preserved [Beck, Gutowski, GP]
- The proof of the homogeneity theorem which states that all backgrounds which preserve more than $1/2$ of supersymmetry are homogeneous Lorentzian spaces [Figueroa-O'Farrill, Hustler]
- The identification of all Killing superalgebras of AdS backgrounds in 10- and 11-dimensions [Beck, Gutowski, Gran, GP]

New developments

The problem has become tractable because of three key recent developments

- The Killing spinor equations (KSEs) of type II 10- and 11-dimensional supergravities have been solved over the AdS subspace for all $AdS_n \times_w M^{D-n}$ backgrounds with the most general allowed fluxes leading to the identification of the number of supersymmetries that can be preserved [Beck, Gutowski, GP]
- The proof of the homogeneity theorem which states that all backgrounds which preserve more than $1/2$ of supersymmetry are homogeneous Lorentzian spaces [Figueroa-O'Farrill, Hustler]
- The identification of all Killing superalgebras of AdS backgrounds in 10- and 11-dimensions [Beck, Gutowski, Gran, GP]

Assumptions

- The fields of warped AdS backgrounds, $\mathrm{AdS}_n \times_w M^{D-n}$, are assumed to be smooth and invariant under the isometries of the AdS subspace. No other assumptions are made on the fields including assumptions on the form of Killing spinors
- Moreover the focus will be on warped AdS solutions with the most general allowed fluxes that admit a compact without boundary internal space M^{D-n}
- The novelty of the approach is that it is completely general

Assumptions

- The fields of warped AdS backgrounds, $\mathrm{AdS}_n \times_w M^{D-n}$, are assumed to be smooth and invariant under the isometries of the AdS subspace. No other assumptions are made on the fields including assumptions on the form of Killing spinors
- Moreover the focus will be on warped AdS solutions with the most general allowed fluxes that admit a compact without boundary internal space M^{D-n}
- The novelty of the approach is that it is completely general

Assumptions

- The fields of warped AdS backgrounds, $AdS_n \times_w M^{D-n}$, are assumed to be smooth and invariant under the isometries of the AdS subspace. No other assumptions are made on the fields including assumptions on the form of Killing spinors
- Moreover the focus will be on warped AdS solutions with the most general allowed fluxes that admit a compact without boundary internal space M^{D-n}
- The novelty of the approach is that it is completely general

D=11 supergravity

D=11 Supergravity: The bosonic fields are the metric g and a 4-form field strength, F, $dF = 0$.
The Einstein field equation of the theory is

$$R_{MN} = \frac{1}{12} F_{ML_1L_2L_3} F_N{}^{L_1L_2L_3} - \frac{1}{144} g_{MN} F_{L_1L_2L_3L_4} F^{L_1L_2L_3L_4} \ .$$

and the field equation of the 4-form field strength is

$$d \star_{11} F - \frac{1}{2} F \wedge F = 0 \ ,$$

The KSE is the vanishing condition of the supersymmetry variation of the gravitino

$$\mathcal{D}_M \epsilon \equiv \nabla_M \epsilon - \left(\frac{1}{288} \Gamma_M{}^{L_1L_2L_3L_4} F_{L_1L_2L_3L_4} - \frac{1}{36} F_{ML_1L_2L_3} \Gamma^{L_1L_2L_3} \right) \epsilon = 0$$

where ϵ is a 32 component Majorana $\mathfrak{spin}(10,1)$ spinor.

- The gravitino KSE is a parallel transport equation. The associated connection has holonomy in a GL group
- The 10-dimensional supergravities have an additional KSE, the dilatino KSE $\mathcal{A}\epsilon = 0$, where $\mathcal{A}$ depends on the fields but it is algebraic in ϵ.
- If there exist a $\epsilon \neq 0$ solution to the KSEs, then the associated solution to the field equations is called supersymmetric.
- The number N of linear independent solutions to the KSEs is the number of supersymmetries preserved by a background.

- The gravitino KSE is a parallel transport equation. The associated connection has holonomy in a GL group
- The 10-dimensional supergravities have an additional KSE, the dilatino KSE $\mathcal{A}\epsilon = 0$, where $\mathcal{A}$ depends on the fields but it is algebraic in ϵ.
- If there exist a $\epsilon \neq 0$ solution to the KSEs, then the associated solution to the field equations is called supersymmetric.
- The number N of linear independent solutions to the KSEs is the number of supersymmetries preserved by a background.

- The gravitino KSE is a parallel transport equation. The associated connection has holonomy in a GL group
- The 10-dimensional supergravities have an additional KSE, the dilatino KSE $\mathcal{A}\epsilon = 0$, where $\mathcal{A}$ depends on the fields but it is algebraic in ϵ.
- If there exist a $\epsilon \neq 0$ solution to the KSEs, then the associated solution to the field equations is called supersymmetric.
- The number N of linear independent solutions to the KSEs is the number of supersymmetries preserved by a background.

AdS backgrounds

The a priori number of sypersymmetries preserved by D=11, IIB and IIA AdS backgrounds are [Beck, Gutowski, GP]

AdS_n	N
$n = 2$	$2k, k \leq 32$
$n = 3$	$2k, k \leq 32$
$n = 4$	$4k, k \leq 8$
$n = 5$	$8, 16, 24, 32$
$n = 6$	$16, 32$
$n = 7$	$16, 32$

Table: The proof for AdS_2 requires an application of Hopf's maximum principle. For the rest, no such assumption is necessary.

Sketching the proof

The warp, flux, AdS backgrounds are special cases of near horizon geometries. For $n > 2$, the metric is

$$ds^2 = 2du(dr + rh) + A^2(dz^2 + e^{2z/\ell}\sum_{a=1}^{n-3}(dx^a)^2) + ds^2(M^{11-n}) ,$$

with

$$h = -\frac{2}{\ell}dz - 2A^{-1}dA ,$$

A is the warp factor and ℓ the AdS radius.

To find the number of supersymmetries preserved

- Solve the KSEs along the lightcone directions (u, r)
- solve the KSEs along z and then the remaining x^a coordinates
- count the multiplicity of Killing spinors

Sketching the proof

The warp, flux, AdS backgrounds are special cases of near horizon geometries. For $n > 2$, the metric is

$$ds^2 = 2du(dr + rh) + A^2(dz^2 + e^{2z/\ell}\sum_{a=1}^{n-3}(dx^a)^2) + ds^2(M^{11-n}) ,$$

with

$$h = -\frac{2}{\ell}dz - 2A^{-1}dA ,$$

A is the warp factor and ℓ the AdS radius.
To find the number of supersymmetries preserved

- Solve the KSEs along the lightcone directions (u, r)
- solve the KSEs along z and then the remaining x^a coordinates
- count the multiplicity of Killing spinors

The solution of the KSEs along the AdS subspace give

$$\begin{aligned}\epsilon &= \sigma_+ + \sigma_- - \ell^{-1} e^{\frac{z}{\ell}} x^a \Gamma_{az}\sigma_- - \ell^{-1} A^{-1} u \Gamma_{+z}\sigma_- \\ &\quad + e^{-\frac{z}{\ell}}\tau_+ - \ell^{-1} A^{-1} r e^{-\frac{z}{\ell}} \Gamma_{-z}\tau_+ - \ell^{-1} x^a \Gamma_{az}\tau_+ + e^{\frac{z}{\ell}}\tau_-\end{aligned}$$

where $\Gamma_\pm\sigma_\pm = \Gamma_\pm\tau_\pm = 0$.

The remaining independent KSEs on the internal space M^{D-n} are

$$D_i^{(\pm)}\sigma_\pm = 0\,, \quad D_i^{(\pm)}\tau_\pm = 0\,,$$

which are the naive restriction of the gravitino KSEs onto the internal space, and

$$\mathcal{A}^{(\pm)}\sigma_\pm = \mathcal{A}^{(\pm)}\tau_\pm = 0\,; \quad \mathcal{B}^{(\pm)}\sigma_\pm = 0\,, \quad \mathcal{C}^{(\pm)}\tau_\pm = 0\,,$$

where $\mathcal{A}^{(\pm)}$ are the naive restrictions of the dilatino KSEs onto the internal space and

- the integration over z introduces new algebraic KSEs denoted by $\mathcal{B}^{(\pm)}$ and $\mathcal{C}^{(\pm)}$

The solution of the KSEs along the AdS subspace give

$$\begin{aligned}\epsilon &= \sigma_+ + \sigma_- - \ell^{-1} e^{\frac{z}{\ell}} x^a \Gamma_{az}\sigma_- - \ell^{-1} A^{-1} u \Gamma_{+z}\sigma_- \\ &\quad + e^{-\frac{z}{\ell}}\tau_+ - \ell^{-1} A^{-1} r e^{-\frac{z}{\ell}} \Gamma_{-z}\tau_+ - \ell^{-1} x^a \Gamma_{az}\tau_+ + e^{\frac{z}{\ell}}\tau_-\end{aligned}$$

where $\Gamma_\pm \sigma_\pm = \Gamma_\pm \tau_\pm = 0$.
The remaining independent KSEs on the internal space M^{D-n} are

$$D_i^{(\pm)}\sigma_\pm = 0\,, \quad D_i^{(\pm)}\tau_\pm = 0\,,$$

which are the naive restriction of the gravitino KSEs onto the internal space, and

$$\mathcal{A}^{(\pm)}\sigma_\pm = \mathcal{A}^{(\pm)}\tau_\pm = 0\,; \quad \mathcal{B}^{(\pm)}\sigma_\pm = 0\,, \quad \mathcal{C}^{(\pm)}\tau_\pm = 0\,,$$

where $\mathcal{A}^{(\pm)}$ are the naive restrictions of the dilatino KSEs onto the internal space and

- the integration over z introduces new algebraic KSEs denoted by $\mathcal{B}^{(\pm)}$ and $\mathcal{C}^{(\pm)}$

The solution of the KSEs along the AdS subspace give

$$\begin{aligned}\epsilon &= \sigma_+ + \sigma_- - \ell^{-1} e^{\frac{z}{\ell}} x^a \Gamma_{az} \sigma_- - \ell^{-1} A^{-1} u \Gamma_{+z} \sigma_- \\ &\quad + e^{-\frac{z}{\ell}} \tau_+ - \ell^{-1} A^{-1} r e^{-\frac{z}{\ell}} \Gamma_{-z} \tau_+ - \ell^{-1} x^a \Gamma_{az} \tau_+ + e^{\frac{z}{\ell}} \tau_-\end{aligned}$$

where $\Gamma_\pm \sigma_\pm = \Gamma_\pm \tau_\pm = 0$.
The remaining independent KSEs on the internal space M^{D-n} are

$$D_i^{(\pm)} \sigma_\pm = 0 \,, \quad D_i^{(\pm)} \tau_\pm = 0 \,,$$

which are the naive restriction of the gravitino KSEs onto the internal space, and

$$\mathcal{A}^{(\pm)} \sigma_\pm = \mathcal{A}^{(\pm)} \tau_\pm = 0 \,; \quad \mathcal{B}^{(\pm)} \sigma_\pm = 0 \,, \quad \mathcal{C}^{(\pm)} \tau_\pm = 0 \,,$$

where $\mathcal{A}^{(\pm)}$ are the naive restrictions of the dilatino KSEs onto the internal space and

- the integration over z introduces new algebraic KSEs denoted by $\mathcal{B}^{(\pm)}$ and $\mathcal{C}^{(\pm)}$

The counting

To count the multiplicity, it turns out that if $\sigma_\pm$ is a solution, so is

$$\tau_\pm = \Gamma_{z\mathrm{a}}\sigma_\pm$$

and vice-versa

Similarly, if σ_+, τ_+ is a solution, so is

$$\sigma_- = A\Gamma_-\Gamma_z\sigma_+ \ , \quad \tau_- = A\Gamma_-\Gamma_z\tau_+$$

and vice-versa.

Furthermore, if σ_+ is Killing spinor, then

$$\sigma'_+ = \Gamma_{\mathrm{ab}}\sigma_+ \ , \quad \mathrm{a} < \mathrm{b} \ ,$$

is also a Killing spinor.

- The number of supersymmetries are derived by counting the linearly independent solutions

Lichnerowicz Theorem

This relates the zero modes of the Dirac operator to parallel spnors. In particular notice that $D^2 = \nabla^2 - \frac{1}{4}R$ where D is the Dirac operator and ∇ is the Levi-Civita connection. Then after a partial integration

$$\int \parallel D\epsilon \parallel^2 = \int \parallel \nabla\epsilon \parallel^2 + \frac{1}{4}\int R \parallel \epsilon \parallel^2$$

- If $R = 0$, all zero modes of the Dirac operator are parallel
- if $R > 0$, the Dirac operator has no zero modes

Some applications include

- Counting problems, ie the number of parallel (Killing) spinors of 8-d manifolds with holonomy strictly $Spin(7)$, $SU(4)$, $Sp(2)$ and $\times^2 Sp(1)$ is given by the index of the Dirac operator
- Necessary conditions for the existence of metrics with $R > 0$

New Lichnerowicz type of theorems

One can establish new Lichnerowicz type theorems ($D = 11$) as

$$\mathscr{D}^{(\pm)}\sigma_\pm = 0 \Longleftrightarrow D_i^{(\pm)}\sigma_\pm = 0 \,, \quad \mathcal{B}^{(\pm)}\sigma_\pm = 0 \,,$$

where $\mathscr{D}^{(\pm)} = \Gamma^i D_i^{(\pm)} + q\mathcal{B}^{(\pm)}$ for some $q \in \mathbb{R}$.
These are based on maximum principle formulae

$$\nabla^2 \parallel \sigma_+ \parallel^2 + nA^{-1}\partial^i A \partial_i \parallel \sigma_+ \parallel^2 = 2\langle \mathbb{D}_i^{(+)}\sigma_+, \mathbb{D}^{(+)i}\sigma_+ \rangle$$
$$+2\frac{9n-18}{11-n} \parallel \mathcal{B}^{(+)}\sigma_+ \parallel^2 \,,$$

which is established after using the field equations, where
$\mathbb{D}_i^{(+)} = D_i^{(+)} + \frac{2-n}{11-n}\Gamma_i \mathcal{B}^{(+)}$.

- If the solution is smooth, the warp factor A is nowhere zero.

New Lichnerowicz type of theorems

One can establish new Lichnerowicz type theorems ($D = 11$) as

$$\mathscr{D}^{(\pm)}\sigma_\pm = 0 \Longleftrightarrow D_i^{(\pm)}\sigma_\pm = 0\,, \quad \mathcal{B}^{(\pm)}\sigma_\pm = 0\,,$$

where $\mathscr{D}^{(\pm)} = \Gamma^i D_i^{(\pm)} + q\mathcal{B}^{(\pm)}$ for some $q \in \mathbb{R}$.
These are based on maximum principle formulae

$$\nabla^2 \parallel \sigma_+ \parallel^2 + nA^{-1}\partial^i A \partial_i \parallel \sigma_+ \parallel^2 = 2\langle \mathbb{D}_i^{(+)}\sigma_+, \mathbb{D}^{(+)i}\sigma_+\rangle + 2\frac{9n-18}{11-n} \parallel \mathcal{B}^{(+)}\sigma_+ \parallel^2\,,$$

which is established after using the field equations, where
$\mathbb{D}_i^{(+)} = D_i^{(+)} + \frac{2-n}{11-n}\Gamma_i\mathcal{B}^{(+)}$.

- If the solution is smooth, the warp factor A is nowhere zero.

Homogeneity

Conjecture: All solutions of a supergravity theory preserving more than half of the supersymmetry are homogenous. [Meessen]

Theorem: All solutions of $D = 11$, IIB and IIA supergravities that preserve strictly more than 16 supersymmetries are homogeneous [Figueroa-O'Farrill, Hustler]

Proof: One can show that given two Killing spinors ϵ_1 and ϵ_2, the 1-form bilinear

$$\langle \epsilon_1, \Gamma_M \epsilon_2 \rangle_D \, dx^M$$

is Killing and leaves all the remaining fields invariant. In the Euclidean case where $\langle \cdot, \cdot \rangle$ is positive definite, the proof simplifies. If the vector bilinears do not span the tangent space of the spacetime there is an X such that

$$X^M \langle \epsilon_1, \Gamma_M \epsilon_2 \rangle = \langle \epsilon_1, \not{X} \epsilon_2 \rangle = 0$$

Thus the spinors $\not{X}\epsilon$ for every Killing spinor ϵ are orthogonal to all Killing spinors, and so

$$\not{X} : \quad \mathcal{K} \to \mathcal{K}^\perp$$

But $\not{X}^2 = |X|^2 \mathbf{1}$ and as $X \neq 0$, the map is an injection. However this cannot be if $\dim \mathcal{K}^\perp < \dim \mathcal{K}$ which is the case for more than 16 supersymmetries. Thus $X = 0$ and the spacetime is homogenous. □

Homogeneity

Conjecture: All solutions of a supergravity theory preserving more than half of the supersymmetry are homogenous. [Meessen]

Theorem: All solutions of $D = 11$, IIB and IIA supergravities that preserve strictly more than 16 supersymmetries are homogeneous [Figueroa-O'Farrill, Hustler]

Proof: One can show that given two Killing spinors ϵ_1 and ϵ_2, the 1-form bilinear

$$\langle \epsilon_1, \Gamma_M \epsilon_2 \rangle_D \, dx^M$$

is Killing and leaves all the remaining fields invariant. In the Euclidean case where $\langle \cdot, \cdot \rangle$ is positive definite, the proof simplifies. If the vector bilinears do not span the tangent space of the spacetime there is an X such that

$$X^M \langle \epsilon_1, \Gamma_M \epsilon_2 \rangle = \langle \epsilon_1, \not{X} \epsilon_2 \rangle = 0$$

Thus the spinors $\not{X}\epsilon$ for every Killing spinor ϵ are orthogonal to all Killing spinors, and so

$$\not{X} : \quad \mathcal{K} \to \mathcal{K}^\perp$$

But $\not{X}^2 = |X|^2 \mathbf{1}$ and as $X \neq 0$, the map is an injection. However this cannot be if $\dim \mathcal{K}^\perp < \dim \mathcal{K}$ which is the case for more than 16 supersymmetries. Thus $X = 0$ and the spacetime is homogenous. $\square$

Homogeneity

Conjecture: All solutions of a supergravity theory preserving more than half of the supersymmetry are homogenous. [Meessen]

Theorem: All solutions of $D = 11$, IIB and IIA supergravities that preserve strictly more than 16 supersymmetries are homogeneous [Figueroa-O'Farrill, Hustler]

Proof: One can show that given two Killing spinors ϵ_1 and ϵ_2, the 1-form bilinear

$$\langle \epsilon_1, \Gamma_M \epsilon_2 \rangle_D \, dx^M$$

is Killing and leaves all the remaining fields invariant. In the Euclidean case where $\langle \cdot, \cdot \rangle$ is positive definite, the proof simplifies. If the vector bilinears do not span the tangent space of the spacetime there is an X such that

$$X^M \langle \epsilon_1, \Gamma_M \epsilon_2 \rangle = \langle \epsilon_1, \not{X} \epsilon_2 \rangle = 0$$

Thus the spinors $\not{X}\epsilon$ for every Killing spinor ϵ are orthogonal to all Killing spinors, and so

$$\not{X} : \quad \mathcal{K} \to \mathcal{K}^{\perp}$$

But $\not{X}^2 = |X|^2 \mathbf{1}$ and as $X \neq 0$, the map is an injection. However this cannot be if $\dim \mathcal{K}^{\perp} < \dim \mathcal{K}$ which is the case for more than 16 supersymmetries. Thus $X = 0$ and the spacetime is homogenous. □

Warping AdS

One of the issues that arise in the classification of warped AdS backgrounds [Gran, Gutowski, GP] is that the metric on AdS_{k+1} can be written as a warped product $AdS_k \times_w \mathbb{R}$

$$ds^2(AdS_{k+1}) = \ell^2 dy^2 + \ell^2 \cosh^2 y \, ds^2(AdS_k) \ , \quad y \in \mathbb{R} \ ,$$

- Any $AdS_n \times_w M^{D-n}$ solution can be re-interpreted as a $AdS_k \times_w M^{D-k}$ solution for $k < n$.
- The Killing spinors of AdS backgrounds do not factorize into Killing spinors on AdS and Killing spinors on the internal space. This is particularly obvious for $\mathbb{R}^k \times_w M^{D-k}$ solutions.
- D=11 supergravity admits $AdS_k \times_w M^{11-k}$ maximally supersymmetric solutions for $k \leq 7$. Similar results apply to other theories.
- There are de Sitter supersymmetric solutions in 10- and 11-dimensional supergravities
- This nesting of warped AdS backgrounds presents one of the difficulties in the classification

Warping AdS

One of the issues that arise in the classification of warped AdS backgrounds [Gran, Gutowski, GP] is that the metric on AdS_{k+1} can be written as a warped product $AdS_k \times_w \mathbb{R}$

$$ds^2(AdS_{k+1}) = \ell^2 dy^2 + \ell^2 \cosh^2 y\, ds^2(AdS_k)\ , \quad y \in \mathbb{R}\ ,$$

- Any $AdS_n \times_w M^{D-n}$ solution can be re-interpreted as a $AdS_k \times_w M^{D-k}$ solution for $k < n$.
- The Killing spinors of AdS backgrounds do not factorize into Killing spinors on AdS and Killing spinors on the internal space. This is particularly obvious for $\mathbb{R}^k \times_w M^{D-k}$ solutions.
- D=11 supergravity admits $AdS_k \times_w M^{11-k}$ maximally supersymmetric solutions for $k \leq 7$. Similar results apply to other theories.
- There are de Sitter supersymmetric solutions in 10- and 11-dimensional supergravities
- This nesting of warped AdS backgrounds presents one of the difficulties in the classification

Warping AdS

One of the issues that arise in the classification of warped AdS backgrounds [Gran, Gutowski, GP] is that the metric on AdS_{k+1} can be written as a warped product $AdS_k \times_w \mathbb{R}$

$$ds^2(AdS_{k+1}) = \ell^2 dy^2 + \ell^2 \cosh^2 y\, ds^2(AdS_k)\;, \quad y \in \mathbb{R}\;,$$

- Any $AdS_n \times_w M^{D-n}$ solution can be re-interpreted as a $AdS_k \times_w M^{D-k}$ solution for $k < n$.
- The Killing spinors of AdS backgrounds do not factorize into Killing spinors on AdS and Killing spinors on the internal space. This is particularly obvious for $\mathbb{R}^k \times_w M^{D-k}$ solutions.
- D=11 supergravity admits $AdS_k \times_w M^{11-k}$ maximally supersymmetric solutions for $k \leq 7$. Similar results apply to other theories.
- There are de Sitter supersymmetric solutions in 10- and 11-dimensional supergravities
- This nesting of warped AdS backgrounds presents one of the difficulties in the classification

Warping AdS

One of the issues that arise in the classification of warped AdS backgrounds [Gran, Gutowski, GP] is that the metric on AdS_{k+1} can be written as a warped product $AdS_k \times_w \mathbb{R}$

$$ds^2(AdS_{k+1}) = \ell^2 dy^2 + \ell^2 \cosh^2 y\, ds^2(AdS_k) , \quad y \in \mathbb{R} ,$$

- Any $AdS_n \times_w M^{D-n}$ solution can be re-interpreted as a $AdS_k \times_w M^{D-k}$ solution for $k < n$.
- The Killing spinors of AdS backgrounds do not factorize into Killing spinors on AdS and Killing spinors on the internal space. This is particularly obvious for $\mathbb{R}^k \times_w M^{D-k}$ solutions.
- D=11 supergravity admits $AdS_k \times_w M^{11-k}$ maximally supersymmetric solutions for $k \leq 7$. Similar results apply to other theories.
- There are de Sitter supersymmetric solutions in 10- and 11-dimensional supergravities
- This nesting of warped AdS backgrounds presents one of the difficulties in the classification

Warping AdS

One of the issues that arise in the classification of warped AdS backgrounds [Gran, Gutowski, GP] is that the metric on AdS_{k+1} can be written as a warped product $AdS_k \times_w \mathbb{R}$

$$ds^2(AdS_{k+1}) = \ell^2 dy^2 + \ell^2 \cosh^2 y \, ds^2(AdS_k) \,, \quad y \in \mathbb{R} \,,$$

- Any $AdS_n \times_w M^{D-n}$ solution can be re-interpreted as a $AdS_k \times_w M^{D-k}$ solution for $k < n$.
- The Killing spinors of AdS backgrounds do not factorize into Killing spinors on AdS and Killing spinors on the internal space. This is particularly obvious for $\mathbb{R}^k \times_w M^{D-k}$ solutions.
- D=11 supergravity admits $AdS_k \times_w M^{11-k}$ maximally supersymmetric solutions for $k \leq 7$. Similar results apply to other theories.
- There are de Sitter supersymmetric solutions in 10- and 11-dimensional supergravities
- This nesting of warped AdS backgrounds presents one of the difficulties in the classification

Killing superalgebras

The Killing superalgebras $\mathfrak{g} = \mathfrak{g}_0 \oplus \mathfrak{g}_1$ with $\mathfrak{g}_0 = \mathbb{R}\langle V_{K_{\mathbf{mn}}}\rangle$ and $\mathfrak{g}_1 = \mathbb{R}\langle Q_{\epsilon_{\mathbf{m}}}\rangle$ of supersymmetric backgrounds are defined as follows [Gauntlett, Myers, Townsend; Figueroa-O'Farrill]:

$$\{Q_{\epsilon_{\mathbf{m}}}, Q_{\epsilon_{\mathbf{n}}}\} = V_{K_{\mathbf{mn}}} \ , \quad [V_{K_{\mathbf{mn}}}, Q_{\epsilon_{\mathbf{p}}}] = Q_{\mathcal{L}_{K_{\mathbf{mn}}}\epsilon_{\mathbf{p}}} \ , \quad [V_{K_{\mathbf{mn}}}, V_{K_{\mathbf{pq}}}] = V_{[K_{\mathbf{mn}}, K_{\mathbf{pq}}]} \ ,$$

where $K_{\mathbf{mn}} = \langle \Gamma_0 \epsilon_{\mathbf{m}}, \Gamma_M \epsilon_{\mathbf{n}}\rangle dx^M$ is the 1-form bilinear, $[K_{\mathbf{mn}}, K_{\mathbf{pq}}]$ is the Lie commutator of two vector fields and

$$\mathcal{L}_X \epsilon = \nabla_X \epsilon + \frac{1}{8} dX_{MN} \Gamma^{MN} \epsilon \ ,$$

is the spinorial Lie derivative of ϵ with respect to the vector field X.

- $V_{K_{\mathbf{mn}}} = V_{\mathbf{mn}}$ are the even generators and $Q_{\epsilon_{\mathbf{m}}} = Q_{\mathbf{m}}$ are the odd ones.

AdS_k Killing Superalgebras

The Killing superalgebras of warped AdS_k, $k > 3$, backgrounds are

AdS_k KSAs in $D = 10$ and $D = 11$

N	AdS_4	AdS_5	AdS_6	AdS_7
4	$\mathfrak{osp}(1\|4)$	-	-	-
8	$\mathfrak{osp}(2\|4)$	$\mathfrak{sl}(1\|4)$	-	-
12	$\mathfrak{osp}(3\|4)$	-	-	-
16	$\mathfrak{osp}(4\|4)$	$\mathfrak{sl}(2\|4)$	$\mathfrak{f}^*(4)$	$\mathfrak{osp}(6,2\|2)$
20	$\mathfrak{osp}(5\|4)$	-	-	-
24	$\mathfrak{osp}(6\|4)$	$\mathfrak{sl}(3\|4)$	-	-
28	$\mathfrak{osp}(7\|4)$	-	-	-
32	$\mathfrak{osp}(8\|4)$	$\mathfrak{sl}(4\|4)/1_{8\times 8}$	-	$\mathfrak{osp}(6,2\|4)$

Table: For AdS_k backgrounds with compact without boundary internal space $\mathfrak{g}_0 = \mathfrak{so}(k-1,2) \oplus \mathfrak{t}_0$. $\mathfrak{f}^*(4)$ is a different real form to $\mathfrak{f}(4)$ which appears in the AdS_3 case.

AdS_3 superalgebras

AdS_3 is locally a group manifold and the Killing superalgebra $\mathfrak{g}$ decomposes as $\mathfrak{g} = \mathfrak{g}_L \oplus \mathfrak{g}_R$.

AdS_3 KSAs in type II and $d = 11$

N_L	$\mathfrak{g}_L/\mathfrak{c}$
$2n$	$\mathfrak{osp}(n\|2)$
$4n, n > 1$	$\mathfrak{sl}(n\|2)$
$8n, n > 1$	$\mathfrak{osp}^*(4\|2n)$
16	$\mathfrak{f}(4)$
14	$\mathfrak{g}(3)$
8	$\mathfrak{D}(2, 1, \alpha)$
8	$\mathfrak{sl}(2\|2)/1_{4\times 4}$

Table: If the internal space is compact without boundary, $(\mathfrak{g}_L/\mathfrak{c})_0 = \mathfrak{so}(1, 2) \oplus \mathfrak{t}_0/\mathfrak{c}$. The may be a central term $\mathfrak{c}$

Isometry algebras of internal space

N	AdS_4	AdS_5	AdS_6	AdS_7
4	0	-	-	-
8	$\mathfrak{so}(2)$	$\mathfrak{u}(1)$	-	-
12	$\mathfrak{so}(3)$	-	-	-
16	$\mathfrak{so}(4)$	$\mathfrak{u}(2)$	$\mathfrak{so}(3)$	$\mathfrak{so}(3)$
20	$\mathfrak{so}(5)$	-	-	-
24	$\mathfrak{so}(6)$	$\mathfrak{u}(3)$	-	-
28	$\mathfrak{so}(7)$	-	-	-
32	$\mathfrak{so}(8)$	$\mathfrak{su}(4)$	-	$\mathfrak{so}(5)$

Table: These algebras must act effectively on the internal spaces of AdS backgrounds

Isometry algebras of internal space for AdS_3

Table: AdS_3 Killing superalgebras in type II and 11D

N_L	$\mathfrak{g}_L/\mathfrak{c}_L$	$(\mathfrak{t}_L)_0/\mathfrak{c}_L$	$\dim \mathfrak{c}_L$
$2n$	$\mathfrak{osp}(n\|2)$	$\mathfrak{so}(n)$	0
$4n,\ n>2$	$\mathfrak{sl}(n\|2)$	$\mathfrak{u}(n)$	0
$8n, n>1$	$\mathfrak{osp}(4\|2n)$	$\mathfrak{sp}(n)\oplus\mathfrak{sp}(1)$	0
16	$\mathfrak{f}(4)$	$\mathfrak{spin}(7)$	0
14	$\mathfrak{g}(3)$	$\mathfrak{g}_2$	0
8	$\mathfrak{D}(2,1,\alpha)$	$\mathfrak{so}(3)\oplus\mathfrak{so}(3)$	0
8	$\mathfrak{sl}(2\|2)/1_{4\times 4}$	$\mathfrak{su}(2)$	≤ 3

Sketch of proof:

These results have been established under the assumptions either that

- the field are smooth and the internal space is compact without boundary

or that

- the even part of the superalgebra decomposes to that of isometries of AdS and those of the internal space, $\mathfrak{g}_0 = \mathfrak{iso}(AdS) \oplus \mathfrak{t}_0$.

As the dependence of the Killing spinors on the AdS coordinates is known some of the (anti-) commutators of the Killing superalgebra can be explicitly calculated. These are

- $\{Q, Q\} = V_{\mathfrak{iso}(AdS)} + V_{\mathfrak{t}_0}$
- $[V_{\mathfrak{iso}(AdS)}, Q]$

The key commutator that remains to be evaluated is $[V_{\mathfrak{t}_0}, Q]$. It turns out that for AdS$_n$, $n \geq 4$, this can also be found uniquely as a consequence of the assumptions above and the super-Jacobi identities.
For AdS$_n$, $n = 2, 3$ this is not the case. However the problem can be still solved as it can be shown to be related to groups acting transitively and effectively on spheres. □

Main points

The main conclusions of the analysis are

- The internal spaces of AdS backgrounds must admit an almost effective action of a group with Lie algebra $\mathfrak{t}_0$
- For solutions that preserve more than half of supersymmetry ($N > 16$) the internal space must admit a transitive and an almost effective action of a group with Lie algebra $\mathfrak{t}_0$

Main points

The main conclusions of the analysis are

- The internal spaces of AdS backgrounds must admit an almost effective action of a group with Lie algebra $\mathfrak{t}_0$
- For solutions that preserve more than half of supersymmetry ($N > 16$) the internal space must admit a transitive and an almost effective action of a group with Lie algebra $\mathfrak{t}_0$

Main points

The main conclusions of the analysis are

- The internal spaces of AdS backgrounds must admit an almost effective action of a group with Lie algebra $\mathfrak{t}_0$
- For solutions that preserve more than half of supersymmetry ($N > 16$) the internal space must admit a transitive and an almost effective action of a group with Lie algebra $\mathfrak{t}_0$

AdS_6 and AdS_7

Th [Figueroa-O'Farrill, GP]: The maximal supersymmetric AdS solutions ($N = 32$) in 10 and 11 dimensions up to a local isometry are as follows .

- $D = 11$: $AdS_4 \times S^7$ and $AdS_7 \times S^4$
- $D = 10$ IIB: $AdS_5 \times S^5$

□

AdS_6 and AdS_7 backgrounds can preserve either 16 or 32 supersymmetries. So for $N > 16$, these solutions must be maximally supersymmetric. Thus

- There are no $N > 16$ AdS_6 supersymmetric solutions
- The $N > 16$ supersymmetric AdS_7 solutions are locally isometric to the maximally supersymmetric $AdS_7 \times S^4$ solution of 11-dimensional supergravity

AdS_6 and AdS_7

Th [Figueroa-O'Farrill, GP]: The maximal supersymmetric AdS solutions ($N = 32$) in 10 and 11 dimensions up to a local isometry are as follows .

- $D = 11$: $AdS_4 \times S^7$ and $AdS_7 \times S^4$
- $D = 10$ IIB: $AdS_5 \times S^5$

□

AdS_6 and AdS_7 backgrounds can preserve either 16 or 32 supersymmetries. So for $N > 16$, these solutions must be maximally supersymmetric. Thus

- There are no $N > 16$ AdS_6 supersymmetric solutions
- The $N > 16$ supersymmetric AdS_7 solutions are locally isometric to the maximally supersymmetric $AdS_7 \times S^4$ solution of 11-dimensional supergravity

AdS_5

AdS_5 backgrounds preserve $8k$ supersymmetries.

Th[Beck, Gutowski, GP]: Let the internal space of AdS_5 backgrounds be compact without boundary.

- There are no solutions which preserve 24 and 32 supersymmetries in (massive) IIA and 11-dimensional supergravities.
- In IIB, all solutions that preserve $N > 16$ supersymmetries are locally isometric to the maximally supersymmetric $AdS_5 \times S^5$ solution

AdS_5

AdS_5 backgrounds preserve $8k$ supersymmetries.

Th[Beck, Gutowski, GP]: Let the internal space of AdS_5 backgrounds be compact without boundary.

- There are no solutions which preserve 24 and 32 supersymmetries in (massive) IIA and 11-dimensional supergravities.
- In IIB, all solutions that preserve $N > 16$ supersymmetries are locally isometric to the maximally supersymmetric $\mathrm{AdS}_5 \times S^5$ solution

AdS_5 , $N = 24$

Th: There are no smooth AdS_5 solutions preserving 24 supersymmetries with compact without boundary internal space in $D = 11$ supergravity.

There are plenty of AdS_5 solutions apart from the IIB $AdS_5 \times S^5$ preserving less supersymmetry. Previous systematic investigations include [Apruzzi, Fazzi, Passias, Tomasiello].

Proof: The background is

$$ds^2 = 2du(dr + rh) + A^2(ds^2 + e^{\frac{2z}{\ell}}(dx^a)^2) + ds^2(M^6) ,$$
$$F = X , \quad h = -\frac{2}{\ell}dz - 2A^{-1}dA .$$

In this case

$$D_i^{(\pm)} = D_i \pm \frac{1}{2}\partial_i \log A - \frac{1}{288}\Gamma_i{}^{j_1\dots j_4}X_{j_1\dots j_4} + \frac{1}{36}X_{ij_1j_2j_3}\Gamma^{j_1j_2j_3}$$
$$\mathcal{B}^{(\pm)} = -\frac{1}{2}\Gamma_z\Gamma^i\partial_i \log A \mp \frac{1}{2\ell}A^{-1} + \frac{1}{288}\Gamma_z\Gamma^{j_1\dots j_4}X_{j_1\dots j_4}$$

AdS_5 , $N = 24$

Th: There are no smooth AdS_5 solutions preserving 24 supersymmetries with compact without boundary internal space in $D = 11$ supergravity.

There are plenty of AdS_5 solutions apart from the IIB $AdS_5 \times S^5$ preserving less supersymmetry. Previous systematic investigations include [Apruzzi, Fazzi, Passias, Tomasiello].

Proof: The background is

$$ds^2 = 2du(dr + rh) + A^2(ds^2 + e^{\frac{2z}{\ell}}(dx^a)^2) + ds^2(M^6) \ ,$$
$$F = X \ , \quad h = -\frac{2}{\ell}dz - 2A^{-1}dA \ .$$

In this case

$$D_i^{(\pm)} = D_i \pm \frac{1}{2}\partial_i \log A - \frac{1}{288}\Gamma_i{}^{j_1\dots j_4}X_{j_1\dots j_4} + \frac{1}{36}X_{ij_1j_2j_3}\Gamma^{j_1j_2j_3}$$
$$\mathcal{B}^{(\pm)} = -\frac{1}{2}\Gamma_z\Gamma^i\partial_i \log A \mp \frac{1}{2\ell}A^{-1} + \frac{1}{288}\Gamma_z\Gamma^{j_1\dots j_4}X_{j_1\dots j_4}$$

AdS_5 , $N = 24$

Th: There are no smooth AdS_5 solutions preserving 24 supersymmetries with compact without boundary internal space in $D = 11$ supergravity.

There are plenty of AdS_5 solutions apart from the IIB $AdS_5 \times S^5$ preserving less supersymmetry. Previous systematic investigations include [Apruzzi, Fazzi, Passias, Tomasiello].

Proof: The background is

$$ds^2 = 2du(dr + rh) + A^2(ds^2 + e^{\frac{2z}{\ell}}(dx^a)^2) + ds^2(M^6) \ ,$$
$$F = X \ , \quad h = -\frac{2}{\ell}dz - 2A^{-1}dA \ .$$

In this case

$$D_i^{(\pm)} = D_i \pm \frac{1}{2}\partial_i \log A - \frac{1}{288}\Gamma_i{}^{j_1\cdots j_4}X_{j_1\cdots j_4} + \frac{1}{36}X_{ij_1j_2j_3}\Gamma^{j_1j_2j_3}$$
$$\mathcal{B}^{(\pm)} = -\frac{1}{2}\Gamma_z\Gamma^i\partial_i \log A \mp \frac{1}{2\ell}A^{-1} + \frac{1}{288}\Gamma_z\Gamma^{j_1\cdots j_4}X_{j_1\cdots j_4}$$

AdS_5 , $N = 24$

Using the gravitino and algebraic KSEs and maximum principle, one can establish that

$$\parallel \sigma_+ \parallel = \text{const}$$

Furthermore

$$W_i = A\langle \sigma_+, \Gamma_{z12i}\sigma_+ \rangle$$

is Killing and leaves the fields invariant. $\mathfrak{t}_0$ is spanned by the W's. Moreover, from the algebraic KSE one has

$$i_W \star_6 X = 6 \parallel \sigma_+ \parallel^2 dA \ ,$$

This implies

$$i_W dA = 0$$

Then the homogeneity theorem gives A constant and $X = 0$. The fluxes vanish and the warp factor field equation cannot be satisfied. □

AdS_4

AdS_4 solutions preserve $4k$ supersymmetries.

Th [Lautz, Haupt, GP]: Let the internal space of AdS_4 backgrounds be compact without boundary.

- There are no AdS_4 solutions in IIB and massive IIA supergravities that preserve $N > 16$ supersymmetries.
- All AdS_4 solutions of 11-dimensional supergravity that preserve $N > 16$ supersymmetries are locally isometric to the maximally supersymmetric $AdS_4 \times S^7$ solution.
- All IIA AdS_4 solutions that preserve $16 < N \leq 24$ supersymmetries are locally isometric to the $AdS_4 \times CP^3$, $N = 24$, solution of IIA supergravity. There are no IIA AdS_4 solutions that preserve 28 and 32 supersymmetries.

Sketching the proof:

- Unlike the AdS_5 case to establish the above theorem one has to investigate in detail the homogeneous G/H spaces with $\mathfrak{Lie}\, G = \mathfrak{t}_0 = \mathfrak{so}(k)$ for $k > 4$

Sketching the proof:

Consider the 11-dimensional case where the internal space is a 7-dimensional homogeneous manifold.
First one establishes that

$$\| \sigma_+ \| = \text{const}$$

and that for $N > 16$ supersymmetries the warp factor A is constant as well.
Therefore

- all $N > 16$ AdS_4 backgrounds are products $\mathrm{AdS}_4 \times M^7$, where M^7 is a homogeneous space admitting a transitive and effective $\mathfrak{so}(k)$ action.
- The proof proceeds with a case by case analysis

□

Table: 7-dimensional compact, simply connected, homogeneous spaces

	$M^7 = G/H$
(1)	$\frac{\text{Spin}(8)}{\text{Spin}(7)} = S^7$, symmetric space
(2)	$\frac{\text{Spin}(7)}{G_2} = S^7$
(3)	$\frac{SU(4)}{SU(3)}$ diffeomorphic to S^7
(4)	$\frac{Sp(2)}{Sp(1)}$ diffeomorphic to S^7
(5)	$\frac{Sp(2)}{Sp(1)_{max}}$, Berger space
(6)	$\frac{Sp(2)}{\Delta(Sp(1))} = V_2(\mathbb{R}^5)$, not spin
(7)	$\frac{SU(3)}{\Delta_{k,l}(U(1))} = W^{k,l}$ k,l coprime, Aloff-Wallach space
(8)	$\frac{SU(2)\times SU(3)}{\Delta_{k,l}(U(1))\cdot(1\times SU(2))} = N^{k,l}$ k,l coprime
(9)	$\frac{SU(2)^3}{\Delta_{p,q,r}(U(1)^2)} = Q^{p,q,r}$ p,q,r coprime
(10)	$M^4 \times M^3,\ M^4 = \frac{\text{Spin}(5)}{\text{Spin}(4)},\ \frac{SU(3)}{S(U(1)\times U(2))},\ \frac{SU(2)}{U(1)} \times \frac{SU(2)}{U(1)}$ $M^3 = SU(2)\ ,\ \frac{SU(2)\times SU(2)}{\Delta(SU(2))}$
(11)	$M^5 \times \frac{SU(2)}{U(1)},\ M^5 = \frac{\text{Spin}(6)}{\text{Spin}(5)},\ \frac{SU(3)}{SU(2)},\ \frac{SU(2)\times SU(2)}{\Delta_{k,l}(U(1))},\ \frac{SU(3)}{SO(3)}$

Classification of $N > 16$ AdS backgrounds

Assuming that the internal space is compact without boundary, a summary of the results so far is as follows

	AdS_4	AdS_5	AdS_6	AdS_7
$N = 20$	$-$			
$N = 24$	*IIA*	$-$		
$N = 28$	$-$			
$N = 32$	$D = 11$	*IIB*	$-$	$D = 11$

AdS_3 and AdS_2

AdS_3 and AdS_2 backgrounds preserve $2k$ supersymmetries. There are several possibilities for the existence of such backgrounds with $N > 16$ supersymmetries. However one finds

Th [Lautz, Haupt, GP]:Let the internal space be compact without boundary. There are no AdS_3 solutions that preserve $N > 16$ supersymmetries in (massive) IIA, IIB and 11-dimensional supergravities

Th [Beck, Gutowski. Gran, GP]: Under the same assumptions, there are no AdS_2 solutions that preserve $N > 16$ supersymmetries in (massive) IIA, IIB and 11-dimensional supergravities

Sketching the proof: The proof is similar to that of AdS_4 case. The main difference is that the group which acts on the internal space may not even be semisimple. □

AdS_3 and AdS_2

AdS_3 and AdS_2 backgrounds preserve $2k$ supersymmetries. There are several possibilities for the existence of such backgrounds with $N > 16$ supersymmetries. However one finds

Th [Lautz, Haupt, GP]:Let the internal space be compact without boundary. There are no AdS_3 solutions that preserve $N > 16$ supersymmetries in (massive) IIA, IIB and 11-dimensional supergravities

Th [Beck, Gutowski. Gran, GP]: Under the same assumptions, there are no AdS_2 solutions that preserve $N > 16$ supersymmetries in (massive) IIA, IIB and 11-dimensional supergravities

Sketching the proof: The proof is similar to that of AdS_4 case. The main difference is that the group which acts on the internal space may not even be semisimple. □

Heterotic

Th: In heterotic theory with $dH = 0$

- There are no AdS_n, $n > 3$, supersymmetric backgrounds
- There are no smooth AdS_2 backgrounds for which the internal space is compact without boundary
- AdS_3 backgrounds preserve 2,4,6 and 8 supersymmetries
- Smooth AdS_3 backgrounds preserving 8 supersymmetries with compact without boundary internal space are locally isometric to either $AdS_3 \times S^3 \times T^4$ or $AdS_3 \times S^3 \times K_3$
- Although there is no classification of all possible backgrounds, there is a clear overview of all possibilities and what equations should be solved to achieve the task.

Heterotic

Th: In heterotic theory with $dH = 0$

- There are no AdS_n, $n > 3$, supersymmetric backgrounds
- There are no smooth AdS_2 backgrounds for which the internal space is compact without boundary
- AdS_3 backgrounds preserve 2,4,6 and 8 supersymmetries
- Smooth AdS_3 backgrounds preserving 8 supersymmetries with compact without boundary internal space are locally isometric to either $AdS_3 \times S^3 \times T^4$ or $AdS_3 \times S^3 \times K_3$
- Although there is no classification of all possible backgrounds, there is a clear overview of all possibilities and what equations should be solved to achieve the task.

Heterotic

Th: In heterotic theory with $dH = 0$

- There are no AdS_n, $n > 3$, supersymmetric backgrounds
- There are no smooth AdS_2 backgrounds for which the internal space is compact without boundary
- AdS_3 backgrounds preserve 2,4,6 and 8 supersymmetries
- Smooth AdS_3 backgrounds preserving 8 supersymmetries with compact without boundary internal space are locally isometric to either $AdS_3 \times S^3 \times T^4$ or $AdS_3 \times S^3 \times K_3$
- Although there is no classification of all possible backgrounds, there is a clear overview of all possibilities and what equations should be solved to achieve the task.

Heterotic

Th: In heterotic theory with $dH = 0$

- There are no AdS_n, $n > 3$, supersymmetric backgrounds
- There are no smooth AdS_2 backgrounds for which the internal space is compact without boundary
- AdS_3 backgrounds preserve 2,4,6 and 8 supersymmetries
- Smooth AdS_3 backgrounds preserving 8 supersymmetries with compact without boundary internal space are locally isometric to either $AdS_3 \times S^3 \times T^4$ or $AdS_3 \times S^3 \times K_3$
- Although there is no classification of all possible backgrounds, there is a clear overview of all possibilities and what equations should be solved to achieve the task.

Heterotic

Th: In heterotic theory with $dH = 0$

- There are no AdS_n, $n > 3$, supersymmetric backgrounds
- There are no smooth AdS_2 backgrounds for which the internal space is compact without boundary
- AdS_3 backgrounds preserve 2,4,6 and 8 supersymmetries
- Smooth AdS_3 backgrounds preserving 8 supersymmetries with compact without boundary internal space are locally isometric to either $AdS_3 \times S^3 \times T^4$ or $AdS_3 \times S^3 \times K_3$
- Although there is no classification of all possible backgrounds, there is a clear overview of all possibilities and what equations should be solved to achieve the task.

Geometry

The geometry of AdS_3 backgrounds is as follows:

N	M^7	B^k	*fibre*
2	G_2	$-$	$-$
4	$SU(3)$	$U(3)$	S^1
6	$SU(2)$	$self-dual-Weyl$	S^3
8	$SU(2)$	$hyper-Kahler$	S^3

Table: The G-structure of M^7 is compatible with a connection with skew-symmetric torsion. For $N = 4, 6, 8$, M^7 is a local (twisted) fibration over a base space B^k with fibre either S^1 or S^3 . The base spaces B are conformally balanced with respect to the associated fundamental forms.

Geometry

The geometry of AdS_3 backgrounds is as follows:

N	M^7	B^k	*fibre*
2	G_2	$-$	$-$
4	$SU(3)$	$U(3)$	S^1
6	$SU(2)$	$self-dual-Weyl$	S^3
8	$SU(2)$	$hyper-Kahler$	S^3

Table: The G-structure of M^7 is compatible with a connection with skew-symmetric torsion. For $N = 4, 6, 8$, M^7 is a local (twisted) fibration over a base space B^k with fibre either S^1 or S^3 . The base spaces B are conformally balanced with respect to the associated fundamental forms.

Conclusion

- AdS backgrounds in 10- and 11-dimensions exhibit some novel geometric features which have led to a generalization of classic results like the Lichnerowicz theorem.
- There is a classification up to local isometry of all smooth AdS backgrounds in 10- and 11-dimensions which preserve more than 16 supersymmetries and have internal space a compact manifold without boundary
- The next few years there will be much progress towards completing this programme for $N \leq 16$ and exploring the applications in a variety of problems in gravity, gauge theory, string theory, AdS/CFT and geometry.

Conclusion

- AdS backgrounds in 10- and 11-dimensions exhibit some novel geometric features which have led to a generalization of classic results like the Lichnerowicz theorem.
- There is a classification up to local isometry of all smooth AdS backgrounds in 10- and 11-dimensions which preserve more than 16 supersymmetries and have internal space a compact manifold without boundary
- The next few years there will be much progress towards completing this programme for $N \leq 16$ and exploring the applications in a variety of problems in gravity, gauge theory, string theory, AdS/CFT and geometry.

Conclusion

- AdS backgrounds in 10- and 11-dimensions exhibit some novel geometric features which have led to a generalization of classic results like the Lichnerowicz theorem.
- There is a classification up to local isometry of all smooth AdS backgrounds in 10- and 11-dimensions which preserve more than 16 supersymmetries and have internal space a compact manifold without boundary
- The next few years there will be much progress towards completing this programme for $N \leq 16$ and exploring the applications in a variety of problems in gravity, gauge theory, string theory, AdS/CFT and geometry.