

# Dynamics of Tethered Satellite Systems: A Challenge for Applied and Numerical Mathematics

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We start with a short introduction into the concept of tethered satellite systems, that is two or more satellites, which are connected by thin long cables, in orbit around a planet. This is a new concept of space flight with a great number of practically important applications. Then we consider modelling, derivation of equations of motion and numerical simulation of the dynamics of a system of two satellites connected by a visco-elastic tether in orbit around the Earth. Specific attention is paid to the variational formulation of the equations of motion of the system which is of variable mass composition. The equations of motion of the system are a set of nonlinear coupled ordinary and partial differential equations. For usually used tethers they have the property to be stiff which creates problems in the numerical simulation. One possibility to overcome these problems is to introduce an alternative set of variables (natural string variables) for the description of the deformation of the tether. In these new variables the equations strongly decouple with respect to the slow and fast motions, present in the stiff system. This is a crucial step for the efficient numerical integration of such a stiff system, after the discretization of the tether in space by Finite Elements or Finite Differences has been performed. Further, various time integrators, especially qualified for stiff systems, are compared. Some selected (partly animated) simulation results concerning motions of the system with variable tether length are presented. For the practically important processes of deployment or retrieval of one satellite from or to another satellite the introduction of stabilizing control is also discussed.

Besides the numerical simulation, which is most important from an industrial mathematics point of view, we consider also two more theoretical questions for a tethered satellite system moving on a circular orbit around the Earth. First, we analyse stability of relative equilibria by means of the energy momentum method and, second, the occurrence of transient chaotic dynamics for certain initial conditions.