Runge-Kutta schemes as exact schemes

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First we consider standard difference schemes for scalar ordinary initial value problems $\dot{u} = f(u)$, $u(0) = u_0 \in \mathbb{R}$. Traditionally, error expansions are used to obtain convergence results for all differential equations. We use these error expansions to find the most general right hand sides $f(u)$ for which a given numerical scheme is exact, i.e. for which all error terms vanish [1], [2].

Then we consider given differential equations and ask which schemes are exact for them. This is the traditional question asked when dealing with exact schemes [3]. Answering this question is essentially the same as finding explicit solutions to the differential equation and thus not possible in general. If, however, a low-dimensional function space is known to contain the solution to be approximated, then a variable-coefficient Runge-Kutta scheme can be constructed which is exact on this function space. This approach introduced by Ozawa [4] is generalized and further exploited.

Exact schemes are of interest in applications. Useful nonstandard schemes for parabolic (Le Roux[5]) and hyperbolic (Kojouharov & Chen [3, chap.2]) PDEs are based on exact schemes for ODEs and for simpler PDEs, respectively.

References

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