

A Note on a Clothoid and the Gamma Function

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The paper presents a proposal for the generalization of a clothoid (the Cornu spiral, transient curve) - a curve in a plane from the family of pseudospirals. Parametric equations of the generalized clothoid were derived, wherefrom the asymptotic points were located, thus the gamma function occurred. On the basis of geometric properties of the considered curves the conclusions were drawn regarding the possibilities of establishing approximate bilateral boundaries of these points. As far as the authors know, in connection with the foregoing it provides a new possibility of interval limitation of the gamma function, i.e., the calculation of values in its basic interval. It is derived on the basis of the obtained inequality

$$\sum_{k=0}^{2m+1} (-1)^k \frac{\left(\left(2n - \frac{1}{2}\right)\pi\right)^{2k+1-\alpha}}{(2k)!(2k+1-\alpha)} < \frac{\pi}{2\Gamma(\alpha) \cos \frac{\alpha\pi}{2}} < \sum_{k=0}^{2m+2} (-1)^k \frac{\left(\left(2n + \frac{1}{2}\right)\pi\right)^{2k+1-\alpha}}{(2k)!(2k+1-\alpha)},$$

or

$$\sum_{k=0}^{2m+1} (-1)^k \frac{(2n\pi)^{2k+2-\alpha}}{(2k+1)!(2k+2-\alpha)} < \frac{\pi}{2\Gamma(\alpha) \sin \frac{\alpha\pi}{2}} < \sum_{k=0}^{2m+2} (-1)^k \frac{\left(\left(2n + 1\right)\pi\right)^{2k+2-\alpha}}{(2k+1)!(2k+2-\alpha)},$$

with regard to the values of parameters m and n .

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