Titles, Topics and Abstracts

Summer School: Algebraic Structures in Quantum Field Theory
August 26-30, 2019, University of Hamburg

Owen Gwilliam:
Holomorphic field theories and their symmetries

When the equations of motion of a field theory are holomorphic in nature, a wealth of tools from complex geometry and representation theory allow us to analyze and quantize it. In these lectures we will introduce several examples, discuss where they come from (e.g., from complex geometry or twists of supersymmetric theories), and develop higher dimensional analogues of classic results of chiral conformal field theories on Riemann surfaces. A central role will be played by homotopical algebra, in the guise of L_∞ algebras and Batalin-Vilkovisky formalism, and we will aim to motivate these ideas with physical examples.

David Jordan:
A unified quantization of character varieties from factorization homology

The G-character variety $Ch_G(X)$ of a space X is the moduli space of G-local systems on X. When G is reductive and S is a surface, $Ch_G(S)$ carries a canonical Poisson bracket, constructed independently by Atiyah-Bott and Goldman. When M is a 3-manifold with boundary S, $Ch_G(M)$ defines a Lagrangian in $Ch_G(S)$. These moduli spaces play a central role in $N = 4,\ d = 4$ super Yang-Mills theory, and in Chern-Simons field theories.

In the 90's and 00's, three strikingly distinct proposals were made to quantize $Ch_G(S)$: Alekseev-Grosse-Schomerus algebras (representation theory), Fock–Goncharov quantum cluster varieties (combinatorics), and Turaev's skein algebras (topology). Each proposal had its pros and cons; the comparison between the different quantizations was never completed, nor was the determination of many basic properties of each quantization.

In these lectures, I will explain how each quantization can be recovered in the framework of factorization homology of surfaces with coefficients in the quantum group associated to G. As applications, I will explain a new proof of (a strengthened form of) the Unicity Conjecture of Bonahon-Wong, and I will prove that skein modules of closed 3-manifolds are finite-dimensional for generic values of the quantization parameter, answering in the affirmative a question of Carrega and Witten. The three comparison results are joint works with Ben-Zvi and Brochier, and with Le, Schrader and Shapiro, and independent work of Cooke. The latter two applications are joint works with Ganev and Safronov, and with Gunningham and Safronov, respectively.

Mikhail Kapranov:
Factorization algebras in algebraic geometry

Factorization algebras were introduced by Beilinson-Drifeld in the algebro-geometric context (for algebraic curves) and by Lurie and Costello-Gwilliam for smooth or topological manifolds. The goal in both cases was to provide an axiomatic framework for various types of quantum field theories.

The lectures will describe some applications of factorization algebras to "outside" problems (i.e., to problems which, a priori, do not involve this concept). One such problem is that of finding the cohomology of the Lie algebra of vector fields on an algebraic variety. The other concerns the cohomological Hall algebra of an algebraic surface. Based on joint works with B. Hennion and E. Vasserot.

Claudia Scheimbauer:
E_n-algebras in mathematical physics

I will first introduce E_n-algebras and discuss how they arise. Examples include (homotopy) algebras, braided monoidal categories such as the category of finite dimensional representations of a reductive algebraic group Rep G or of the associated quantum group Rep $U_q(\mathfrak{g})$ — which will appear in David Jordan's course, and, coming from topology, n-fold loop spaces. We will see a different incarnation thereof, in terms of factorization algebras, which will also appear in Owen Gwilliam's course.

Then we will discuss Atiyah and Segal's axiomatic approach to topological and conformal quantum field theories, which provides a beautiful link between the geometry of "spacetimes" (cobordisms) and algebraic structures. Combining this with the physical notion of "locality" led to the introduction of the language of higher categories into the topic. Using factorization algebras, we will discuss a particular suitable target, namely the higher Morita category; and its operatic analog due to Haugseng.

Then we turn to discussing the Cobordism Hypothesis — the seminal theorem in the study of extended TFTs — and dualizability. We will discuss dualizability in several different settings, for us most importantly in the higher Morita category. The pictures which appear strongly resemble string diagrams and correspond to algebraic manipulations. Moreover, this will relate to dualizability results by Douglas-Schommer-Pries-Snyder and Brochier-Jordan-Snyder.

If time permits, we will also discuss a relative version of functorial field theories and give a classification thereof. This will allow to construct simple examples using the dualizability results we have seen: the goal is to showcase low-dimensional field theories which are "relative" to their observables.

Masahito Yamazaki:
Integrability and Perturbative Quantum Field Theory

In a celebrated paper in 1989, E. Witten explained knot invariants from the three-dimensional Chern-Simons theory. Since there are some parallels between knots and integrable models, one can ask if there is also a gauge theory explanation of the integrable models. Recently it has been found (by K. Costello, E. Witten and myself) that the gauge theory in question is a four-dimensional analog of the Chern-Simons theory, which is partly topological and partly holomorphic. In this lecture we explain how the perturbative analysis of this quantum field theory explains many of the properties of integrable models, including the spectral parameter and the infinite-dimensional symmetry algebra.