Optimization of Complex Systems – 7th Exercise Sheet.

Discussion of the solutions in the exercise on December 16, 2019.

Problem 1 (formal Lagrange technique): Derive the candidate of the adjoint equation for the problem of Homework 6/3 by the formal Lagrange technique.

Problem 2 (necessary optimality conditions): Let a bounded Lipschitz domain $\Omega \in \mathbb{R}^N$ with boundary Γ and functions $y_\Omega \in L^2(\Omega)$, $e_\Omega \in L^2(\Omega)$, and $e_\Gamma \in L^2(\Gamma)$ be given and assume that the control $u \in L^2(\Omega)$. Derive the necessary optimality conditions for the problem

$$\min J(y, u) := \frac{1}{2} \int_\Omega |y - y_\Omega|^2 \mathrm{d}x + \int_\Gamma e_\Gamma y \mathrm{d}s + \frac{1}{2} \int_\Omega |u|^2 \mathrm{d}x$$

$$\text{subject to } -\Delta y = u + e_\Omega \text{ in } \Omega,$$

$$\frac{\partial y}{\partial n} = 0 \text{ on } \Gamma,$$

$$0 \le u(x) \le 1 \text{ a.e. in } \Omega.$$