Universität Hamburg
Fachbereich Mathematik
Dr. Matthias Voigt

Summer Term 2019

Model Reduction Homework Sheet 6.

The problems will be discussed in the exercise on Thursday, June 11.

Problem 1: (Balanced truncation). Go to the course webpage and download the benchmark models `CDplayer.mat`, `iss.mat`, and `iss2.mat`.

a) Use the MATLAB function `balancmr` to reduce the models `CDplayer.mat` and `iss.mat` to the reduced order $r = 20$ (Alternatively, you can implement a MATLAB function by your own. Cholesky factors of Lyapunov equation solutions can be computed with the function `lyapchol`.).

- Compare your results to the ones obtained by modal truncation. For this, generate the sigma plots of the reduced models and their error transfer functions.
- Compute the $\mathcal{H}_\infty$ error bounds for modal truncation and balanced truncation and compare them to the true errors.

b) Now apply the low-rank Cholesky factor ADI method for balanced truncation on the "large-scale" model `iss2.mat`.

- Go to `http://www.mpi-magdeburg.mpg.de/projects/mess/` and download the `M-M.E.S.S.` zip archive. Get familiar with the demos in the subfolder `DEMOS/` and reduce the model to order $r = 30$ with an ADI residual tolerance of 10^{-4}.
  Remark: You may want to check out the `sssMOR` package, see `https://www.rt.mw.tum.de/?sssMOR` which provides a nice graphical user interface to solve the above mentioned tasks.
- Let $P \approx LL^\mathsf{T}$ and $Q \approx RR^\mathsf{T}$ be the Gramian approximations and let $L^\mathsf{T}R = U\Sigma V^\mathsf{T}$ be an SVD with $\Sigma = \operatorname{diag}(\sigma_1, \ldots, \sigma_k)$. Compute an approximate error bound by estimating the uncomputed Hankel singular values $\sigma_{k+1}, \ldots, \sigma_n$ by σ_k. Compare this to the true error bound by computing the full Cholesky factors of P and Q using `lyapchol`.

Problem 2: (Krylov subspaces). Let $F \in \mathbb{C}^{n\times n}$ and $v \in \mathbb{C}^n$ be given. Define the *Krylov subspace*

$$\mathcal{K}_j(F, v) := \operatorname{span}\left\{v, Fv, \ldots, F^{j-1}v\right\}.$$

a) Show that if $F^{m-1}v \neq 0$ and $F^m v = 0$ for some $m \in \mathbb{N}$, then

$$\dim \mathcal{K}_j(F, v) = j, \quad j = 1, \ldots, m.$$

b) Show that $\mathcal{K}_j(F, v) = \mathcal{K}_j(F - \lambda I_n, v)$ is satisfied for all $\lambda \in \mathbb{C}$.

Problem 3: (Moment matching for DAEs). Consider the linear *differential-algebraic* control system

$$\begin{aligned} \tfrac{\mathrm{d}}{\mathrm{d}t}Ex(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) + Du(t), \end{aligned} \tag{1}$$

where E, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{p\times n}$, and $D \in \mathbb{R}^{p\times m}$. Moreover, assume that the matrix pencil $sE - A$ is regular, i. e., $\det(sE - A)$ is not the zero polynomial. Show that the transfer function of (1) given by $G(s) := C(sE - A)^{-1}B + D$ decomposes as

$$G(s) = G_{\mathrm{sp}}(s) + G_{\mathrm{poly}}(s),$$

where $G_{\mathrm{sp}}(s) \in \mathbb{R}(s)^{p\times m}$ is the strictly proper part and $G_{\mathrm{poly}}(s) \in \mathbb{R}[s]^{p\times m}$ is the polynomial part. Derive explicit expressions for G_{sp} and G_{poly}. For this you can make use of the *quasi-Weierstraß form* of the matrix pencil $sE - A$, that is, there exist invertible matrices W, $T \in \mathbb{R}^{n\times n}$ such that

$$W(sE - A)T = s\begin{bmatrix} I_r & 0 \\ 0 & E_{22} \end{bmatrix} - \begin{bmatrix} A_{11} & 0 \\ 0 & I_{n-r} \end{bmatrix},$$

where $E_{22} \in \mathbb{R}^{(n-r)\times(n-r)}$ is nilpotent with index of nilpotency ν. What are the Markov parameters of G?