Research Statement - Lóránt Szegedy

Functorial Quantum Field Theories

My research is about mathematical structures around low dimensional functorial field theories. These are symmetric monoidal functors from a category of n-dimensional bordisms to a symmetric monoidal category, e.g. the category of vector spaces. This formulation is a systematic way of assigning objects (vector spaces) to $(n-1)$-dimensional manifolds and morphisms (linear maps) to n-dimensional manifolds with specified boundary. Functorial field theories are interesting from both a mathematical and a physical point of view: they produce invariants of manifolds compatible with cutting and glueing, and they axiomatise locality of the path integral.

r-Spin Topological Field Theories – previous results

In two dimensions there is a natural generalisation of spin structures using the r-spin group – the r-fold cover of SO_2. Topological field theories with this tangential structure have been considered in [Nov15] and based on this we gave a state sum r-spin TFT computing the Arf invariant and we explicitly described the action of mapping class groups on r-spin surfaces [RS18b]. Building on this and work of [KST] I classified invertible r-spin TFTs with arbitrary target category [Sze19] – they are generated by the TFTs computing the Euler characteristic and the Arf invariant. Understanding this classification is important in physics as invertible field theories are expected to correspond to topological phases of matter. A classification of open-closed r-spin TFTs and their state sum construction is currently work in progress [SS].

Fully extended r-spin TFTs

State sum TFTs correspond to fully extended TFTs [Dav11], the latter assign quantities to arbitrary codimensional manifolds and hence can be phrased as functors between bicategories [BD95] or $(\infty, 2)$-categories [Lur09]. The Cobordism Hypothesis (CH) states that fully extended framed TFTs correspond to fully dualisable objects of the target ($W \in \mathcal{C}^{\mathrm{fd}}$) and r-spin TFTs to homotopy fixed points of the r-spin group. In the bicategorical setting the CH has been proven [Pst14, HV19] and it was shown that the SO_2 action is given by Serre automorphisms S_W and its homotopy fixed points are trivialisations of S_W. Therefore for r-spin TFTs we need to consider trivialisations of S_W^r.

Framed and oriented TFTs with target bicategory $\mathcal{LG}^{\mathrm{gr}}$ graded Landau-Ginzburg (LG) models $\mathcal{LG}^{\mathrm{gr}}$ (with potentials as objects and morphism categories of matrix factorisations) has been given in [CMM18]. There it was shown that every object $W \in \mathcal{LG}^{\mathrm{gr}}$ is fully dualisable and S_W is a degree shifted unit 1-morphism, and potentials W whose zero locus is Calabi-Yau (CY) determine oriented TFTs. **I intend to prove that potentials with fractional CY zero locus determine r-spin TFTs.** Another attractive target for fully extended TFTs is the Morita bicategory of dg-algebras (dgas) [Joh08], where the fully dualisable objects are smooth and proper dgas [Lur14]. **I would like to find when S_A^r for a smooth and proper dga A is trivialisable,** for $r = 1$ this is when A is CY. Both of these examples should yield **non-semisimple fully extended TFTs** which are almost absent in the literature.

The above two bicategories are related: Topologically twisted SUSY sigma models determine A_∞-categories [Car09, KKS14] (which can be strictified to dg-categories). This suggest naturally to consider fully extended TFTs in the $(\infty, 2)$-categorical setting. Since the CH is not completely proven here [Lur09, AF17], I will first try to construct **TFTs with values in the Morita$(\infty, 2)$-category of E_1-algebras** in a symmetric monoidal $(\infty, 1)$-category [Sch14, Hau17] (e.g. the nerve of an A_∞-category [Fao17]). Another target I would like consider, and first construct is an **$(\infty, 2)$-lift of $\mathcal{LG}^{\mathrm{gr}}$** [DM12]. Then **I will try to develop tools to compute homotopy fixed points of SO_2 actions on $(\infty, 2)$-categories,** which is of independent interest.

2-dimensional $\mathcal{N} = 2$ SUSY QFTs and their defect bicategories

Spin surfaces correspond to $\mathcal{N} = 1$ [Sac09] and spin^c surfaces to $\mathcal{N} = 2$ super Riemann surfaces [Wit12]. Therefore in order to study 2d $\mathcal{N} = 2$ supersymmetric (SUSY) TFTs it is useful to understand a combinatorial model of spin^c surfaces [Bud13] that then can be used for a state sum construction and extend the results of [Laz01, MS06, SS].

The combinatorial model of spin surfaces [NR15b, Nov15] was used in [NR15a] to construct functorial QFTs on spin surfaces from oriented functorial defect QFTs [CMS16], in particular from rational CFTs [FRS02, FFRS09]. In this project **I intend to construct 2d $\mathcal{N} = 2$ SUSY CFTs from oriented defect CFTs and compute their bicategory of topological defects** [DKR11]. It will be also interesting to see the relation of these results to the notions of [StTe11]. A motivation for doing this is to be able to compare the defect bicategory $\mathcal{LG}^{\mathrm{gr}}$ of Landau-Ginzburg models [BHLS03, BrRo95, CM16] to the defect bicategory of $\mathcal{N} = 2$ SUSY CFTs that could not have been computed yet. A first comparison has been done using a bosonic version of the defect category in [DCR14]. It will be also interesting to see how this relates to the fully extended $(\infty, 2)$-TFT described above.

3- and 4-dimensional Spin TFTs

Three-dimensional TFTs have been gaining popularity because of their possible application in topological quantum computing therefore it is certainly of physical as well as of mathematical interest to develop the theory of three-dimensional spin TFTs which can describe fermionic as well as bosonic matter.

It is desirable to find the combinatorial data (**super spherical category**) for a state sum construction of three-dimensional spin TFTs [GK16] generalizing the Turaev-Viro model [TV92] based on a combinatorial model of spin manifolds [Bud13]. Another possible approach to 3d spin TFTs is via the surgery description of 3d spin manifolds of [KM99]. This would generalise the Reshetikhin-Turaev construction [RT91] and would take as input data a **super modular tensor category**. Then it will be interesting to compare the resulting TFTs with fully extended TFTs [SP13] or the construction of [BGK17].

Defect lines in LG models are expected to be related to surface defects in a three-dimensional Rozansky-Witten theory [KRS09]. It will be interesting to see how this correspondence can be made precise in the functorial field theory language and with the three-dimensional spin TFTs.

Another direction is to consider four-dimensional spin TFTs which generalise the Crane-Yetter construction [BGIM04, BB17]. The main motivation for this is to understand **defects in four dimensional spin TFTs** and to see which of these could be **new observables in physics**.

Area-dependent Field Theories

Area-dependent quantum field theories (aQFTs) are defined on surfaces with area. These are very similar to two-dimensional TFTs, but they allow infinite dimensional spate spaces. We have classified them in [RS18a], where we also gave a state sum construction of aQFTs with defects – extending aQFTs to stratified surfaces with area – from algebraic data that we call regularised Frobenius algebras and their bimodules. We have shown that 2-dimensional Yang-Mills theory (2dYM) with gauge group G and with Wilson lines as defects [Wit91] is such an aQFT with defects, given by the state sum on the regularised Frobenius algebra $L^2(G)$. We have found invertible defect lines corresponding to outer automorphisms $\mathrm{Out}(G)$ and in [MSS19] we computed the orbifold theory [CR16] of this symmetry, which is 2dYM with gauge group $G \rtimes \mathrm{Out}(G)$. We furthermore gave a conjecture on the volume of the moduli space of flat G-bundles twisted by an $\mathrm{Out}(G)$ bundle computed by 2dYM with the symmetry defects, extending the conjecture of [Wit91].

Deformations of 2dYM

In the physics literature several 1-parameter deformations – so-called q-deformations – of 2dYM have been considered and they appear as dimensional reductions of various quantum field theories, [BuRo95, Kli01, Tac12, SzTi13]. Most of these deformations are related to changing the gauge group G to a quantum group $U_q(g)$. In [BuRo95, Tac12] the area of the surfaces is considered and in e.g. [SzTi13], in the case q is a root of unity, a truncation of q-deformed 2dYM is related to 3-dimensional Chern-Simons theory (3dCS) on S^1-bundles over surfaces.

The first aim of this project is to **construct q-deformed 2dYM theories as aQFTs as a state sum from $U_q(g)$ treated as a regularised Frobenius algebra**. This shall shed more light on the connection of 2dYM and 3dCS and might be used to understand relations to knot invariants and knot homology [AS13]. The second aim is to study the **connection of different q-deformed 2dYM theories** – in particular that of [Kli01] – **to the deformation quantisation** of certain Lie-Poisson manifolds. Finally it would be interesting to understand certain refinements of q-deformed 2dYM called (q, t)-deformations, which are related to Macdonald polynomials [SzTi13].

Continuous Orbifolds

The orbifold of a bulk quantum field theory with a finite group symmetry G is now a standard construction [DVVV89]. To determine the so called twisted sectors of the theory, one needs to average over all group actions, which boils down to summations over G. In [GS12, FR12] limits of minimal models in CFT and orbifolds with compact Lie groups, called continuous orbifolds, have been studied. Here the averaging procedure is more subtle as one need to sum over a compact Lie group.

The generalized orbifold theory [BCP14] is the value of a defect theory on surface with a fine enough defect network labeled by the "group algebra of G". The main idea of this project is to find a way to regulate these infinite summations with an area parameter. More precisely, as 2dYM can be thought of as the orbifold of the trivial theory with compact Lie group, **the continuous orbifold of a CFT should be the orbifold using a regularised Frobenius algebra in the defect category of the CFT**. As the resulting theory should be a CFT, it should be given by zero area limits.

Hamburg, October 30, 2019

References

[AF17] D. Ayala and J. Francis, *The cobordism hypothesis.* [1705.02240 [math.AT]].

[AS13] M. Aganagic and K. Schaeffer, *Refined black hole ensembles and topological strings.* J. High Energy Phys **060** (2013) 1301, [1210.1865 [hep-th]].

[BB17] M. Bärenz and J.W. Barrett, *Dichromatic state sum models for four-manifolds from pivotal functors.* Comm. Math. Phys. **360** (2017) 663–714, [1601.03580 [math-ph]].

[BCP14] I. Brunner, N. Carqueville, and D. Plencner, *A quick guide to defect orbifolds.* Proc. Symp. Pure Math. **88** (2014) 231–242, [1310.0062 [hep-th]].

[BD95] J.C. Baez and J. Dolan, *Higher-dimensional Algebra and Topological Quantum Field Theory.* J. Math. Phys. **36** (1995) 6073–6105, [q-alg/9503002 [math.QA]].

[BGIM04] J.W. Barrett, J.M. Garcia-Islas, and J.F. Martins, *Observables in the Turaev-Viro and Crane-Yetter models.* J. Math. Phys. **48** (2004) 093508, [math/0411281 [math.QA]].

[BGK17] L. Bhardwaj, D. Gaiotto, and A. Kapustin, *State sum constructions of spin-TFTs and string net constructions of fermionic phases of matter.* JHEP **04** (2017) 096, [1605.01640 [cond-mat.str-el]].

[BHLS03] I. Brunner, M. Herbst, Wolfgang Lerche, and Bernhard Scheuner, *Landau-Ginzburg Realization of Open String TFT.* JHEP **0611** (2003) 043, [hep-th/0305133].

[BuRo95] E. Buffenoir and Ph. Roche, *Two-dimensional lattice gauge theory based on a quantum group.* Comm. Math. Phys. **170** no. 3, (1995) 669–698. http://projecteuclid.org/euclid.cmp/1104273256.

[BrRo95] I. Brunner and D. Roggenkamp, *B-type defects in Landau-Ginzburg models.* JHEP **08** (2007) 093, [0707.0922 [hep-th]].

[Bud13] R. Budney, *Combinatorial spin structures on triangulated manifolds.* [1306.4841 [math.GT]].

[Car09] N. Carqueville, *Matrix factorisations and open topological string theory.* JHEP **07** (2009) 005, [0904.0862 [hep-th]].

[CM16] N. Carqueville and D. Murfet, *Adjunctions and defects in Landau-Ginzburg models.* Adv. Math. **289** (2016) 480–566, [1208.1481 [math.AG]].

[CMM18] N. Carqueville and F. Montiel Montoya, *Extending Landau-Ginzburg models to the point.* [1809.10965 [math.QA]].

[CMS16] N. Carqueville, C. Meusburger, and G. Schaumann, *3-dimensional defect TQFTs and their tricategories.* [1603.01171 [math.QA]].

[CR16] N. Carqueville and I. Runkel, *Orbifold completion of defect bicategories.* Quantum Topol. **7** (2016) 203–279, [1210.6363 [math.QA]].

[Dav11] O. Davidovich. *State sums in two dimensional fully extended topological field theories.* PhD thesis, University of Texas at Austin, 2011.

[DCR14] A. Davydov, A.R. Camacho, and I. Runkel, $\mathcal{N} = 2$ *minimal conformal field theories and matrix bifactorisations of* x^d. [1409.2144 [math.QA]].

[DKR11] A. Davydov, L. Kong, and I. Runkel, *Field theories with defects and the centre functor.* Proc. Symp. Pure Math. **83** (2011) 71–130, [1107.0495 [math.QA]].

[DM12] T. Dyckerhoff and D. Murfet, *The Kapustin-Li formula revisited.* Adv. Math. **231** (2012) 1858–1885, [1004.0687 [math.AG]].

[DVVV89] R. Dijkgraaf, C. Vafa, E. Verlinde, and H. Verlinde, *The operator algebra of orbifold models.* Comm. Math. Phys. **123** (1989) 485–526.

[Fao17] G. Faonte, *Simplicial nerve of an A-infinity category.* Theory and Applications of Categories **32** (2017) 31–52, [1312.2127 [math.AT]].

[FFRS09] J. Fröhlich, J. Fuchs, I. Runkel, and C. Schweigert. *Defect lines, dualities, and generalised orbifolds.* In P. Exner, editor, *Proceedings of the XVI International Congress on Mathematical Physics*, pages 608–613. World Scientific, 2009, [0909.5013 [math-ph]].

[FR12] S. Fredenhagen and C. Restuccia, *The geometry of the limit of $N = 2$ minimal models.* JHEP **2012** (2012) 141, [1208.6136 [hep-th]].

[FRS02] J. Fuchs, I. Runkel, and C. Schweigert, *TFT construction of RCFT correlators I: Partition functions.* Nucl. Phys. **B646** (2002) 353–497, [hep-th/0204148].

[GK16] D. Gaiotto and A. Kapustin, *Spin TQFTs and fermionic phases of matter.* Int. J. Mod. Phys. A **31** (2016) 1645044, [1505.05856 [cond-mat.str-el]].

[GS12] M.R. Gaberdiel and P. Suchanek, *Limits of minimal models and continuous orbifolds.* JHEP **1203** (2012) 104, [1112.1708 [hep-th]].

[Hau17] R. Haugseng, *The higher Morita category of E_n-algebras.* Geom. Topol. **21** (2017) 1631–1730, [1412.8459 [math.AT]].

[HV19] J. Hesse and A. Valentino, *The Serre automorphism via homotopy actions and the Cobordism Hypothesis for oriented manifolds.* Cahiers de Topologie et Géométrie Différentielle Catégoriques **LX** (2019) 194–236, [1701.03895 [math.QA]].

[Joh08] N. Johnson, *Morita Theory For Derived Categories: A Bicategorical Perspective.* [0805.3673 [math.AT]].

[KKS14] M. Kapranov, M. Kontsevich, and Y. Soibelman, *Algebra of the infrared and secondary polytopes.* [1408.2673 [math.SG]].

[Kli01] C. Klimčík, *The formulae of Kontsevich and Verlinde from the perspective of the Drinfel'd double.* Commun. Math. Phys. **217** (2001) 203–228, [hep-th/9911239].

[KM99] R. Kirby and P. Melvin. *Canonical framings for 3-manifolds.* In *Proceedings of 6th Gökova Geometry-Topology Conference*, volume 23, pages 89–115, 1999, [math/9903056 [math.GT]].

[KRS09] A. Kapustin, L. Rozansky, and N. Saulina, *Three-dimensional topological field theory and symplectic algebraic geometry I.* Nucl. Phys. B **816** (2009) 295–355, [0810.5415 [hep-th]].

[KST] M. Kreck, S. Stolz, and P. Teichner. *Invertible topological field theories are SKK invariants.* (To appear).

[Laz01] C.I. Lazaroiu, *On the structure of open-closed topological field theory in two dimensions.* Nucl.Phys.B **603** (2001) 497–530, [hep-th/0010269 [hep-th]].

[Lur09] J. Lurie, *On the Classification of Topological Field Theories.* Current Developments in Mathematics **2008** (2009) 129–280, [0905.0465 [math.CT]].

[Lur14] J. Lurie. *Higher Algebra.* (2014).

[MS06] G.W. Moore and G. Segal, *D-branes and K-theory in 2D topological field theory.* [hep-th/0609042].

[MSS19] L. Müller, R.J. Szabo, and L. Szegedy, *Symmetry defects and orbifolds of two-dimensional Yang-Mills theory.* [1907.04734 [hep-th]].

[Nov15] S. Novak. *Lattice topological field theories in two dimensions.* PhD thesis, Universität Hamburg, 2015, http://ediss.sub.uni-hamburg.de/volltexte/2015/7527.

[NR15a] S. Novak and I. Runkel, *Spin from defects in two-dimensional quantum field theory.* [1506.07547 [hep-th]].

[NR15b] S. Novak and I. Runkel, *State sum construction of two-dimensional topological quantum field theories on spin surfaces.* J. Knot Theor. Ramif. **24** (2015) 1550028, [1402.2839 [math.QA]].

[Pst14] P. Pstrągowski, *On dualizable objects in monoidal bicategories, framed surfaces and the Cobordism Hypothesis.* [1411.6691 [math.AT]].

[RS18a] I. Runkel and L. Szegedy, *Area-dependent quantum field theory with defects.* [1807.08196 [math.QA]].

[RS18b] I. Runkel and L. Szegedy, *Topological field theory on r-spin surfaces and the Arf invariant.* [1802.09978 [math.QA]].

[RT91] N. Reshetikhin and V.G. Turaev, *Invariants of 3-manifolds via link polynomials and quantum groups.* Invent. Math. **103** no. 3, (1991) 547–597.

[Sac09] C. Sachse. *Global Analytic Approach to Super Teichmüller Spaces.* PhD thesis, Universität Leipzig, 2009, [0902.3289 [math.AG]].

[Sch14] C. Scheimbauer. *Factorization Homology as a Fully Extended Topological Field Theory.* PhD thesis, ETH Zürich, 2014.

[SP13] C. Schommer-Pries, *Dualizability in Low-Dimensional Higher Category Theory.* [1308.3574 [math.AT]].

[SS] W.H. Stern and L. Szegedy. *Open-closed r-spin topological field theory.* Work in progress.

[StTe11] S. Stolz and P. Teichner. *Supersymmetric field theories and generalized cohomology.* In H. Sati and U. Schreiber, editors, *Mathematical Foundations of Quantum Field Theory and Perturbative String Theory*, volume 83, pages 279–340. AMS, 2011, [1108.0189 [math.AT]].

[SzTi13] R.J. Szabo and M. Tierz, *q-deformations of two-dimensional Yang-Mills theory: Classification, categorification and refinement.* Nucl. Phys. B **876** (2013) 234–308, [1305.1580 [hep-th]].

[Sze19] L. Szegedy, *On invertible 2-dimensional framed and r-spin topological field theories.* [1907.09428 [math.AT]].

[Tac12] Y. Tachikawa, *4d partition function on $S^1 \times S^3$ and 2d Yang-Mills with nonzero area.* Prog. Theor. Exp. Phys. **2013** (2012) , [1207.3497 [hep-th]].

[TV92] V. Turaev and O.Y. Viro, *State sum invariants of 3-manifolds and quantum 6jsymbols.* Topology **31** no. 4, (1992) 865–902.

[Wit91] E. Witten, *On quantum gauge theories in two dimensions.* Commun. Math. Phys. **141** (1991) 153–209.

[Wit12] E. Witten, *Notes On Super Riemann Surfaces And Their Moduli.* [1209.2459 [hep-th]].