Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

CHAMPP
CENTER IN HAMBURG FOR
ASTRO-, MATHEMATICAL AND
PARTICLE PHYSICS

LECTURE COURSE IN THE QUANTUM UNIVERSE RESEARCH SCHOOL

Winter Term 2020/2021

Enumerative Geometry

Helge Ruddat

Course Description:

Hermann Schubert developed an effective method for solving counting problems in geometry in his book *Kalkül der abzählenden Geometrie* that appeared in 1879. A basic example is the question *how many lines in* $\mathbb{R}^3$ *meet four given lines in general position?*. Do you know the answer? The calculus is now known as *Schubert calculus*, see (1),(2),(3). Schubert calculus is immensely powerful, e.g. you can use it to find that the number of twisted cubic curves in 3-space that are tangential to 12 quadric surfaces is 5,819,539,783,680. Even though it clearly worked, for a long time it has been unknown what the theoretical basis for the calculus truly is, i.e. nobody knew why it worked. It became the famous Hilbert problem 15 to find a theoretical basis for Schubert calculus.

This was looong time ago. Today we know that intersection theory, the spaces of stable maps and Gromov-Witten theory form the right theoretical framework that underlies Schubert calculus - though there are also competing alternative approaches like Donaldson-Thomas theory. The purpose of this lecture is to introduce the basics of Gromov-Witten theory with a particular view to mirror symmetry and thus the relationship with tropical curve counting.

Prerequisites:

When taking this class, you should have some basic understanding of algebraic geometry or complex geometry or complex analysis. That is, you should either know what an algebraic variety is or know what a complex manifold is. These two notions converge nicely into the notion of compact Riemann surface alias projective algebraic curve which is the one-dimensional version of each. We will talk a lot about compact Riemann surfaces alias projective curves. This class is new in Hamburg and will be tailored in real time to suit whoever is attending this lecture.

Literature:

(1) https://en.wikipedia.org/wiki/Schubert_calculus
(2) *Kalkül der abzählenden Geometrie* von Hermann Schubert, Springer Berlin Heidelberg, 384S.
(3) *Problem 15. Rigorous foundation of Schubert's enumerative calculus*, by Steven Kleiman, in F.Browder Hilberts problems, Proceedings of the Symposium in Pure Mathematics of the American Mathematical Society, Held at Northern Illinois University 1974, S. 445-482.
(4) *Moduli of Curves* by Joe Harris and Ian Morrison
(5) *Notes on stable maps and quantum cohomology*, Rahul Pandharipande and William Fulton, 1995
(6) *Mirror Symmetry and Algebraic Geometry* by David Cox and Sheldon Katz, 1998
(7) *B-Model Gromov-Witten Theory* by Emily Clader, Yongbin Ruan
(8) *An Invitation to Quantum Cohomology, Kontsevich's Formula for Rational Plane Curves* by Kock, Vainsencher
(9) *Intersection Theory with Applications to the Computation of Gromov-Witten Invariants*, Dissertation Hieb Dang: https://kluedo.ub.uni-kl.de/frontdoor/deliver/index/docId/3750/file/Hiep_Dang_thesis.pdf

Date and Place: Tue 16:15–17:45, Wed 16:15–17:45
Zoom: https://uni-hamburg.zoom.us/j/5406259591?pwd=VjlCZnJ2cjBodEdHNlFSbFVJYUF4dz09
Meeting-ID: 540 625 9591
Kenncode: 732543

Problem Classes: Thu 10:15–11:45
BigBlueButton: https://mathbbb.physnet.uni-hamburg.de/b/tim-iqn-cmc-cpq

Starting on: 3 November 2020