UNIVERSITEIT VAN AMSTERDAM
INSTITUTE FOR LOGIC, LANGUAGE AND COMPUTATION

Axiomatic Set Theory

(Axiomatische Verzamelingentheorie)

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Homework Set # 12

Exercise 12.1 (Normal Measures)
Recall that we call an ultrafilter U on κ a **normal measure** if for every $X \in U$ and every regressive function $f : X \to \kappa$ there is a $Y \in U$ such that f is constant on Y.
Show that an ultrafilter is a normal measure if and only if it is closed under diagonal intersections.

Exercise 12.2 (Consistency strength of some large cardinal axioms)
Let (M) be the statement "there is a measurable cardinal", let $(\mathsf{M}+\mathsf{I})$ be the statement "there is a measurable cardinal κ and an inaccessible cardinal $\lambda > \kappa$", and let $(2\mathsf{M})$ be the statement "there are two different measurable cardinals".
Prove that

(1) $\mathsf{ZFC} + (\mathsf{M}+\mathsf{I})$ proves the consistency of $\mathsf{ZFC} + (\mathsf{M})$, *i.e.*, "there is a set X such that $X \models \mathsf{ZFC} + (\mathsf{M})$".
(2) $\mathsf{ZFC} + (2\mathsf{M})$ proves the consistency of $\mathsf{ZFC} + (\mathsf{M}+\mathsf{I})$.

Hint. The proofs are very similar to the proof of Theorem 12.12 in Jech's book: cut off the universe at the right point (*i.e.*, take $\mathbf{V}_\alpha$ for the proper choice of α) and prove that the remaining rest is the object you want. For the second claim, think of Lemma 10.21 in Jech.