Seminar on symplectic geometry

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Summary of topics

(1) **Linear symplectic geometry** [1, p. 38-47], [2, p. 38-47]

- symplectic vector spaces, symplectic complement of a subspace, the different types of linear subspaces and their normal forms
- linear symplectic reduction
- relations between $\mathrm{Sp}(2n)$, $GL(n,\mathbb{C})$ and $O(2n)$
- homotopy equivalence to $U(n)$
- fundamental group
- suggested exercisees: 2.1.2, 2.1.14, 2.2.11

(2) **Basics of symplectic manifolds** (2 sessions?) [1, p. 94-108], [2, p. 81-93]

- basic definitions
- symplectic diffeomorphisms, symplectic vector fields
- Hamiltonian vector fields and Hamiltonian flows, Poisson brackets
- Hamiltonian isotopies and the group $\mathrm{Ham}(M,\omega)$
- more examples: Kodaira-Thurston manifold, cotangent bundles
- suggested exercises: 3.1.4., 3.1.14., 3.1.16., 3.1.21.

(3) **Moser's argument with applications** [1, p. 108-112], [2, p. 93-98]

- Moser's argument and Moser isotopies
- Darboux' theorem
- Moser stability
- suggested exercise: 3.2.8.

(4) **Symplectic vector bundles** [1, p. 79-88 and 91-93], [2, p. 68-77]

- definitions, basic examples
- compatible complex structures
- unitary trivializations
- first Chern class
- suggested exercises: 2.7.6., 2.7.7. (assuming Theorem 2.7.5.)

(5) **Kähler manifolds** [1, p. 153 and 161-172], [2, p. 117-118 and 123-132]

- definition of compatible almost complex structure
- Nijenhuis tensor and integrability
- Kähler manifolds
- the example $\mathbb{C}P^n$ in detail

(6) **Special submanifolds and neighborhood theorems**
[1, p. 116-122], [2, p. 99-104]

- types of submanifolds and examples
- *optional:* Maslov class (needs discussion of Maslov index as in [1, p. 47-54], [2, p. 48-54])
- neighborhood theorems:
  - in differential topology
  - for symplectric submanifolds
  - for Lagrangian submanifolds

(7) **Hamiltonian group actions and moment maps** (2 sessions)
[1, p. 191-195 and 202-213], [2, p. 151-154 and 161-170]

- Hamiltonian circle actions and reduction in the free case
- Hamiltonian action of a general Lie group: definition and basic characterization
- formulation in terms of the moment map
- examples

(8) **Symplectic reduction** [1, p. 218-229], [2, p. 173-179]

- isotropic foliation of coisotropic submanifolds
- the Marsden-Weinstein quotient
- examples

(9) **The nonsqueezing theorem and its significance** [1, p. 458-465], [2, p. 371-380]

- statement of Gromov's nonsqueezing theorem
- definition of symplectic capacities
- C^0-rigidity of the symplectomorphism group

(10) **Summary of proof of nonsqueezing using J-holomorphic curves**

References

[1] D. McDuff and D. Salamon, *Introduction to Symplectic Topology*, Oxford University Press, Third Edition, 2017

[2] D. McDuff and D. Salamon, *Introduction to Symplectic Topology*, Oxford University Press, Second Edition, 1998