Research Summary

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Higher Gauge Theory and Differential Cohomology

In recent years, my research focussed on the mathematical foundations of abelian higher gauge theories and their quantization. In joint work with C. Bär, A. Schenkel and R. Szabo, we investigate various aspects of differential cohomology, using the original Cheeger-Simons model of differential characters [8] and its relative version [5].

This work culminated in a Lecture Notes volume [2] on *Differential Characters*. The book covers both classical and new results. Especially we prove existence and uniqueness of fiber integration by geometric methods. Moreover, we study relative differential cohomology, a differential refinement of relative or mapping cone cohomology. We prove that it is endowed with a unique natural structure of a module over the absolute differential cohomology ring.

Relative differential characters are best understood as a generalization of bundle gerbes [11] and bundle 2-gerbes [14] to arbitrary degree. In [2], [3] we show that bundle (2-)gerbes with connection over a fixed submersion $\pi : Y \to X$ are classified up to isomorphism (preserving $\pi : Y \to X$) by relative characters in $\widehat{H}^n(\pi;\mathbb{Z})$, where $n = 3,4$.

We construct explicit examples of relative differential characters, which we call *Cheeger-Chern-Simons characters* [3]. They combine well-known differential characteristic classes [8] with the corresponding Chern-Simons forms [9]. Cheeger-Chern-Simons characters generalize the so-called Chern-Simons bundle 2-gerbe [7] to arbitrary degree. We use Cheeger-Chern-Simons characters in [3] to derive a notion of higher degree analogues of differential String classes [15, 16] – differential trivializations of arbitary universal characteristic classes. We recover the canonical 3-forms [13] that are present in differential String classes.

Moreover we investigate in [3] the differential cohomology of (compact, connected) Lie groups. We derive splitting results for the differential cohomogies of G and BG and obtain a Hopf theorem in differential cohomology. We also establish explicit descriptions of transgression maps and a differential cohomology version of Borel transgression theorem.

Another application of ordinary differential cohomology is to the quantization of abelian (higher) gauge theories. In [4] we construct from the differential cohomology on globally hyperbolic Lorentzian manifolds an algebraic quantum field theory, more precisely: a locally covariant quantum field theory [6]. Actually, our functor only satisfies the causality and time slice axioms, but violates the locality axiom.

In the near future, we plan to systematically investigate (jointly with K. Waldorf) the differential cohomology of étale or Lie groupoids. These allow to treat the geometry and differential topology of group actions, orbifolds and foliations on equal footing. We expect that most of the above mentioned results generalize to this setting.

INFINITE DIMENSIONAL GEOMETRY

Currently, I am working on particular problems in infinite dimensional geometry: the geometry of principal String bundles. The String group is a 3-connected cover of the Spin group (resp. of a compact, simple, simply connected Lie group). It can be realized as an infinite dimensional Fréchet Lie group [12].

In work in progress, joint with C. Wockel, we study so-called String structues, i.e. lifts of the structure group in principal bundles from Spin to String. We construct connections on String structures, using the so-called *Caloron correspondence*, due to Hekmati-Murray-Vozzo [10]. We aim at a correspondence between String structures with connections (together with additional data), and String connections in the sense of K. Waldorf [15]. Moreover, we try to construct the corresponding Chern-weil and Chern-Simons forms in the spirit of S. Rosenberg & Co.

CLASSICAL ABELIAN GAUGE THEORIES

In less recent years, my work centered around classical abelian gauge theories, notably Seiberg-Witten theory – a nonlinear $\mathrm{U}(1)$ gauge theory. I studied the geometry of the Seiberg-Witten moduli space in the natural L^2-metric [1]. Similar research for Yang-Mills moduli spaces was undertaken by Groisser-Parker, L. Habermann, Hitchin, and others.

We intend to undertake a new approach to classical nonlinear abelian gauge theories in the near future.

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NILPOTENT LIE GROUPS AND UNITARY GROUPS

DANIEL BELTIŢĂ

My research interests belong to finite- and infinite-dimensional Lie theory and representation theory. I am currently interested particularly in: 1. nilpotent Lie groups, and 2. unitary groups of operator algebras. Both of them share some features with the compact Lie groups. Here are some recent references on these topics:

1. **Representations of nilpotent Lie groups and algebras**

1.1. Topology of unitary dual spaces of nilpotent Lie groups

[1] I. Beltiţă, D. Beltiţă, J. Ludwig, *Fourier transforms of C^*-algebras of nilpotent Lie groups.* Preprint arXiv:1411.3254 [math.OA].
[2] I. Beltiţă, D. Beltiţă, *Coadjoint orbits of stepwise square integrable representations.* Preprint arXiv:1408.1857 [math.RT].

1.2. Structure of nilpotent Lie algebras

[1] I. Beltiţă, D. Beltiţă, *On Kirillov's lemma for nilpotent Lie algebras.* J. Algebra **427** (2015), 85–103.
[2] I. Beltiţă, D. Beltiţă, *Faithful representations of infinite-dimensional nilpotent Lie algebras.* Forum Math. **27** (2015), no. 1, 255–267.

1.3. Pseudo-differential Weyl calculus on coadjoint orbits

[1] I. Beltiţă, D. Beltiţă, *Boundedness for Weyl-Pedersen calculus on flat coadjoint orbits.* Int. Math. Res. Not. IMRN (to appear).
[2] I. Beltiţă, D. Beltiţă, *Algebras of symbols associated with the Weyl calculus for Lie group representations.* Monatsh. Math. **167** (2012), no. 1, 13–33.
[3] I. Beltiţă, D. Beltiţă, *Modulation spaces of symbols for representations of nilpotent Lie groups.* J. Fourier Anal. Appl. **17** (2011), no. 2, 290319.
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2. **Representations of unitary groups of operator algebras**

2.1 Classification of representations

[1] D. Beltiţă, K.-H. Neeb, *Nonlinear completely positive maps and dilation theory for real involutive algebras.* Preprint arXiv:1411.6398 [math.OA].
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2.2. Geometric realizations of representations

[1] D. Beltiţă, J.E. Galé, *Linear connections for reproducing kernels on vector bundles.* Math. Z. **277** (2014), no. 1–2, 29–62.
[2] D. Beltiţă, J.E. Galé, *Universal objects in categories of reproducing kernels.* Rev. Mat. Iberoam. **27** (2011), no. 1, 123–179.
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2.2. Classical groups related to operator ideals

[1] D. Beltiţă, S. Patnaik, G. Weiss, *Cartan subalgebras of operator ideals.* Preprint arXiv: 1408.4897 [math.OA].
[2] D. Beltiţă, *Iwasawa decompositions of some infinite-dimensional Lie groups.* Trans. Amer. Math. Soc. **361** (2009), no. 12, 6613–6644.

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Singular Symplectic Fréchet Reduction of Yang-Mills theory

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Keywords: Symplectic Geometry, Global Analysis, Gauge Theory, Mathematical Physics

Background

Actions of infinite-dimensional Lie groups on infinite-dimensional manifolds occur everywhere in global analysis. For example, the diffeomorphism group of a finite-dimensional manifold acts on the space of Riemannian metrics or on the space of vector fields. Such group actions are not just interesting and natural from a mathematical point of view, but also have diverse application in physics ranging from fluid mechanics to shape analysis. Gauge theory provides a further example. There, the group of gauge transformations acts on the space of connections. While the general theory of infinite-dimensional locally convex Lie groups is rather well-understood (see for example the review [Nee06]), very little is known about their actions on infinite-dimensional manifolds beyond a case-by-case study. I am interested in the geometric and topological structure of the orbit space of an infinite-dimensional Lie group action. In particular, I would like to better understand the case where the action is not free and thus the orbit space is not a smooth manifold, but a stratified space. So far the physical effects caused by these singularities are not well understood. In the case of Yang–Mills theory, recent research [Hei+90; RSV02; HRS09] suggests that the singular structure is closely related to non-perturbative phenomenons like quark confinement.

Slice Theorem for Fréchet Lie Group Actions

Slices provide a valuable tool to investigate group actions. They reduce a G-action on a manifold M to an action of the stabilizer subgroup on some invariant submanifold. In particular, the existence of a slice at every point of M guarantees that the orbit space M/G is stratified by smooth manifolds. So, in this case, the quotient M/G is a collection of manifolds that fit nicely together. As such, a slice yields a concrete description of the singular strata.

A slice theorem for Fréchet Lie group actions has been known for some time in special cases. So, for example, the action of gauge transformations on the space of connections [ACM89] as well as the action of the diffeomorphism group on the space of Riemannian metrics [Bou75] admits slices. First approaches towards a more general study were made by Subramaniam [Sub84]. In my Master thesis [Die13], I generalized the aforementioned works and expanded them to a systematic study of Fréchet Lie group actions. With the help of the Nash–Moser theorem, I was able to generalize the finite-dimensional result of Palais [Pal61] under some extra assumptions to a general slice theorem in the Fréchet context.

Singular Symplectic Fréchet Reduction

Consider a finite-dimensional symplectic manifold (M, ω) acted upon by a Lie group G. Symplectic reduction provides a way to pass to a suitable quotient space so that the quotient inherits a symplectic structure. If the reduction is carried out at regular values of the momentum map J, then the quotient $J^{-1}(0)/G$ is a smooth symplectic manifold. However, often the group action is not free and thus the quotient is only a stratified symplectic space.

Many examples from global analysis naturally fit into this general framework. For instance, the space of connections of a principal bundle carries a symplectic structure, which is invariant under the action of gauge transformations. In this case, the momentum map is given by the curvature of a connection and thus the symplectic quotient $J^{-1}(0)/G$ coincides with the space of equivalence classes of flat connections. Atiyah and Bott [AB83] studied this moduli space over a Riemann surface by formally using ideas from symplectic geometry. It is, therefore, of interest to generalize the symplectic reduction scheme to the infinite-dimensional Fréchet setting. To do this in a rigorous and not just formal way is the final goal of my PhD project.

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Research summary
Nora Ganter

This program is about *categorical groups*, a.k.a. *2-groups* or *group-like categories*. The notion of categorical group is a refinement of the notion of group, in which the symmetries themselves are related by symmetries. A number of important groups naturally occur in the form of 2-groups, and we plan to show that studying their previously ignored categorical aspects leads to simplification of some important and difficult mathematics. Preliminary work suggests that our program leads to an entirely new and illuminating development of the representations of affine Lie algebras, and we also propose to give a simple construction of the conjectural *Monster 2-group*, which sheds light on the most difficult aspects of the Monster, including triality.

We judge our success by how simple and natural we can keep our constructions: in the ideal scenario, we would like them to turn out simpler than their known, non-categorical counterparts. We are guided by the philosophy that, just as a group is often best understood as the symmetries of a naturally occuring object, a categorical group is often best understood as the *refined symmetries* of a higher categorical object, i.e., an object whose symmetries have symmetries of their own.

The program falls into two parts: 1. constructing categorical groups and 2. studying their actions as refined symmetries.

(1) We plan to give new constructions of some prominent categorical groups. Our approach is constructive, low-tech, and deliberately avoids obstruction theoretic arguments. Projects:
   (a) Construct the *String 2-groups* as the refined symmetries of string theories, namely of the Wess-Zumino-Witten models. This is mostly done, there is a pre-release preprint, joint with Matthew Ando. Our methodology is different from Fiorenza, Rogers and Schreiber's. The most important idea is the *thin bordism chain complex* of a smooth manifold M, providing a strikingly simple model for the cobordism 2-groupoid of an extended sigma-model with target space M.
   (b) In recent work, available online, we gave a very simple, hands-on construction of *2-group extensions of tori*.
   (c) Give a simple construction of the conjectural *Monster 2-group* as a refined symmetry group. In previous work, we encountered strong empirical evidence for this conjecture of Mason's. Work in progress constructs such a refinement of $2^{1+24} \cdot Co_0$ as the symmetries of a representation involving the categorical Leech torus, see Point 2(a).

(2) We plan to systematically develop the representation and character theory of categorical groups. Others have attempted to approach this question via loop group representations. We emphasise that our formalism is quite simple and works without ever referring to loops or infinite dimensional manifolds. In fact,

   our preliminary results strongly suggest that the representation theory of Lie 2-groups gives an entirely new geometric counterpart to the representation theory of affine Lie algebras, which is analytically much less intricate than loop group representations.

(a) Representations of categorical tori. In work in progress, I show that categorical tori are realized quite naturally as functors and natural transformations action on $\mathcal{C}oh_{T_{\mathbb{C}}}$, the category of coherent sheaves on the complexified torus. If $\mathcal{T}$ is the categorical Leech torus, then one can make sense of the basic representation of $\mathcal{T}$ and of the semidirect product $\mathcal{T} \rtimes \{pm1\}$, Point 1(c) refers to the symmetries of this basic representation. This will be the topic of my talk.

(b) Categorical class functions of categorical tori. Preliminary results: starting only from the idea of examining what a class function (in the sense of Bartlett-Ganter-Kapranov-Willerton) of a torus 2-group looks like, we recover key features of the representation theory of loop groups, namely the Looijenga line bundle, the theta function formalism and, conjectureally, the Verlinde fusion product. Their derivation is straigh-forward from our results in 1.(b).

(c) Representation and character theory of finite categorical groups. This was the subject of the 2013 Masters thesis of Ganter's student Robert Usher. In joint work with Usher, we are planning to further develop this theory and to study applications, e.g., to generalized moonshine.

(d) Weyl 2-groups and the characters of general Lie 2-groups. Planning stage. We have the relevant definitions in place. There is a straight-forward approach to analyzing these objects, building on our results in Part 1(b). This project is likely to have applications to Kapranov's conjectural *super-duper symmetry* formalism.

Long-term interests include *the categorical aspects of McKay correspondence.*

1 Research summary for Hendrik Grundling

My research is mainly in Mathematical Physics, centered on mathematical problems which originate in Quantum Field Theory. I have worked on quantum constraints, obstructions to quantization, quantum gauge theory and the C*-algebras required to model quantum field theories. However, the strand of research which brings me to this workshop is a sequence of projects to analyze & construct group C*-algebras for groups which are not locally compact, and crossed products for singular actions of topological groups on C*-algebras, i.e. where the actions are not strongly continuous, or the groups are not locally compact. These are joint projects with Karl-Hermann Neeb.

Research Summary

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My interest in infinite-dimensional structures / geometry focusses on mapping "spaces" in supergeometry and their applications to (classical and quantum) field theory, PDE-theory for fermionic quantities and possible applications to geometry.

In [4], we construct an object representing the "space" $\underline{SC}^{\infty}(X,Y)$ of all "morphism" $X \to Y$ between arbitrary, finite-dimensional supermanifolds X, Y within the functorial approach to supergeometry ([7],[9]), which has already been successfully applied to construct diffeomorphism supergroups [10]. This object is a functor from the category of (finite-dimensional) Grassmann algebras into a suitable category of (possibly infinite-dimensional) manifolds, which is defined by an exponential-law-type relation. Remarkably, the resulting structure turns out to be "only" a locally affine (in the sense of Michor [6]) supermanifold in case X is not compact. This somewhat unsatisfactory feature emerges from a conflict between the functoriality requirement and the standard construction of manifolds of mappings, modelled on spaces of compactly supported sections. Since all reasonable (super-)spacetimes in physical theories are non-compact, this problem will be addressed again in future research.
Equivalently, the structure of $\underline{SC}^{\infty}(X,Y)$ can be characterised making use of a component-field-type decomposition of its elements, thus basically reducing the problem to one of ordinary ∞-dim. geometry (in preparation).

In joint work with T.P.Hack and A.Schenkel ([3]), we have applied super mapping spaces to construct (free) supersymmetric field theories in the locally covariant sense ([1]), i.e. a functor from a certain category of super spacetimes to a category of algebras of observables. As was to be expected from results in the physics literature, the commonly used concept of morphisms between supermanifolds proves insufficient to capture supersymmetry transformations and hence to describe supersymmetric field theories. We show that an enrichment of the category of super spacetimes, over the category of functors from Grassmann algebras to sets, resolves the issue. In essence, we replace the morphism sets by the larger mapping space objects discussed above, which are functors of the aforementioned type. Even though we do not yet make use of any notion of smoothness on $\underline{SC}^{\infty}(X,Y)$ in [3], we expect that this extra structure (esp. generalizations of micro-local analytic concepts) will give some insight in the finiteness properties of supersymmetric QFTs.

In current work in progress, joint with I.Khavkine, we try to understand fermionic or mixed classical field theories within the framework of covariant phase space (see e.g. [5] and references given there). Roughly speaking, the phase space $\mathcal{P}$ is given by the (∞-dim.) space of solutions of classical equations of motion and symplectic/Poisson structures are obtained through Peierl's construction ([8]). Thus, our goal is to understand fermionic (quasilinear) hyperbolic PDE and their spaces of solutions in terms of ∞-dim. supergeometry and give a rigorous construction of the symplectic structure, which is usually only described in a formal way in the physics literature. We believe that this possible at least in the purely fermionic situation, the presence of non-linear Bosonic contributions and/or gauge degrees of freedom clearly presenting additional difficulties. Eventually, we also hope to clarify the relation of the induced Poisson algebra structure on $C^{\infty}(\mathcal{P})$ to the one obtained from the "off-shell" formalism (see e.g. [2]). In the latter approach, an algebra structure is constructed on $C^{\infty}(\mathrm{allclassicalfields})$

(i.e. "off-shell") at the first place and the equations of motion are taken into account in a second step, by dividing by the ideal generated by them.

To give a brief perspective on possible future research: It should be interesting to relate parts of the results described above to the construction of (supersymmetric) path integrals and also to constructions in Batalin-Vilkovisky formalism. Moreover, algebraic structures arising during the construction of super mapping spaces may be related to structures appearing in theory of renormalisation.

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THE ATIYAH ALGEBROID OF THE CHERN-SIMONS BUNDLE 2-GERBE

PEDRAM HEKMATI

Let $P \to M$ be a principal circle bundle. The automorphisms of P are the diffeomorphisms of P that commute with the action of the circle group $\mathbb{T}$. We have a short exact sequence of Fréchet Lie groups,

$$1 \to C^\infty(M, \mathbb{T}) \to \mathrm{Aut}(P) \to \mathrm{Diff}_P(M) \to 1 \tag{0.1}$$

where $\mathrm{Diff}_P(M)$ denotes the subgroup of diffeomorphisms of M that preserve the isomorphism class of P.

At the Lie algebra level, (0.1) corresponds to the functor $C^\infty(M, \cdot)$ applied to the Atiyah sequence,

$$0 \to M \times \mathbb{R} \to TP/\mathbb{T} \to TM \to 0$$

Next, let $\mathcal{G}$ be a bundle gerbe on M. For each $\phi \in \mathrm{Diff}(M)$, an automorphism of $\mathcal{G}$ is defined as a stable isomorphism $\hat{\phi}\colon \mathcal{G} \to \phi^*\mathcal{G}$ that lifts ϕ. In `arXiv:1108.1525`, it is shown that the collection of all such lifts is a coherent 2-group $\mathrm{Aut}(\mathcal{G})$ and we have an exact sequence of 2-groups,

$$1 \to \mathrm{Pic}(M) \to \mathrm{Aut}(\mathcal{G}) \to \mathrm{Diff}(M) \to 1 \tag{0.2}$$

Using a Čech approach, the infinitesimal version of (0.2) can be described as a strict extension of Lie 2-algebras, where the middle term is isomorphic to an exact Courant algebroid. In this sense, the latter plays the role of the Atiyah algebroid for gerbes.

Problems:

(1) Elucidate the relation between the above-described ‘higher’ Atiyah algebroid and the one appearing in `arXiv:1009.2975`.

(2) Describe the Atiyah algebroid of the Chern-Simons bundle 2-gerbe (see `arXiv:math/0004117`) using heterotic Courant algebroids, introduced in `arXiv:1308.5159` (and discussed in my talk).

Research

Bas Janssens

Gauge groups arise as transformations of gauge theories, such as (Q)ED, (Q)CD and WZW-models. Over the last years, I have studied several aspects of their representation theory: I classified their bounded unitary representations (with K.-H. Neeb), I classified their central extensions (with C. Wockel), and I studied the technically challenging, but much more relevant, theory of *unbounded projective unitary representations* of general infinite dimensional Lie groups (also with K.-H. Neeb). Building on these results, I plan to bring more structure to the representation theory of gauge groups, enriching a field that is known for its many scattered examples with a more systematic approach. Together with K.H. Neeb, I have just obtained the first success in this direction: classifying of the unbounded projective unitary representations which are of *positive energy*. This extends the classification of unitary highest weight representations of affine Kac-Moody algebras, which are, roughly, gauge groups over the circle.

Historically, **String geometry** emerged from the work of Killingback and Witten on integrality properties of what is now called the Witten genus. Recently, Waldorf has managed to define a *fusion structure* on a certain U(1)-bundle over the frame bundle of loop space. I plan to develop a conformal net model of the string 2-group, and use this to extend the fusion structure to the spinor bundle over loop space. Such a fusion structure is believed to be connected to $\mathrm{Diff}(S^1)$-equivariance of Witten's 'Dirac operator on loop space'. I have given several presentations about these ideas in conferences (Bonn, Greifswald), but also in meetings of the *String Geometry Network*, of which I am a member.

Lie algebra sheaves (LAS) are the protagonists in Singer-Sternberg's interpretation of the foundational work of Lie and Cartan. Their work is devoted to the 'transitive case' (in the sense that the corresponding pseudogroup induces only one orbit), which is a quite severe restriction, because many interesting examples are not transitive. There is some literature in the 'intransitive case', but only some very mild intransitivity is allowed (one requires that the orbits are the fibers of a submersion!), so that even very simple examples are excluded. Together with M. Crainic, I aim to provide a theory of LAS that is fully adapted to the non-transitive case. At the same time, I plan to develop a suitable cohomology theory in this more general context, based on ideas from my work with Wockel on cohomology of gauge algebras (these are examples of LAS!), combined with Lie pseudogroup techniques (Spencer cohomology). The motivation comes from the relevance of LA cohomology in geometry and mathematical physics, and there is a large list of examples and possible applications, such as gauge algebras, Lie algebras of vector fields, sections of Lie algebroids, and infinitesimal symmetries of G-structures.

Ralf Meyer

Summary of current research interests:

- K-theory and KK-theory for C^*-algebras
- bicategories of C^*-algebras with homomorphisms or correspondences as arrows, actions of groups, crossed modules, etc., on C^*-algebras and crossed modules for those
- analysis and homological algebra for bornological algebras and modules

Research Interests
Professor Michael Murray

• *The general caloron correspondence* Pedram Hekmati, Michael K. Murray and Raymond F. Vozzo. Journal of Geometry and Physics **62**(2), (2012), 224–241. `arXiv:1105.0805`

In its original form the caloron correspondence related instantons on $\mathbb{R}^3 \times S^1$ to Bogomolny monopoles on $\mathbb{R}^3$. This can be generalised to describe a correspondence between G bundles on $M \times S^1$ and LG bundles on M where LG is the loop group of smooth maps from S^1 into G. In this paper we describe the most general version of this correspondence which relates particular G bundles on the total space of a fibration $Y \to M$ with fibre X to infinite-dimensional bundles on M whose structure group is the gauge group of a particular G bundle on X. These results are used to define characteristic classes of gauge group bundles. Explicit but complicated differential form representatives are computed in terms of a connection and Higgs field.

• *On the existence of bibundles.* Michael Murray, David Michael Roberts and Danny Stevenson. Proceedings of the London Mathematical Society, **105**(6), (2012), 1290–1314. `arXiv:1102.4388`

This paper concerns bibundles which are principal right G bundles with a commuting left G action. (We actually consider a slightly more complicated situation using crossed-modules.) Such a bundle has a so-called *type map* $\phi\colon M \to \mathrm{Out}(G)$ which is a rigid invariant of the bundle in the sense that isomorphic bibundles have the same type maps. This is an important constraint on the existence of bibundles which we explain and also feeds into the classifying theory of bibundles which we describe. There are close relationships with loop group bundles.

• *The Faddeev-Mickelsson-Shatashvili anomaly and lifting bundle gerbes.* Pedram Hekmati, Michael K. Murray, Danny Stevenson and Raymond F. Vozzo, Communications in Mathematical Physics, **319**(2), (2013), 379–393. `arXiv:1112.1752`

In unpublished work I have been thinking about bundles and bundle gerbes where the group that acts changes isomorphically from fibre to fibre. This is closely related to groupoids. In this paper we use this theory and that of the caloron transform to study the Faddeev-Mickelsson-Shatashvili (FSM) anomaly. In gauge theory, the FSM anomaly arises as a prolongation problem for the action of the gauge group on a bundle of projective Fock spaces. We study this anomaly from the point of view of bundle gerbes and give several equivalent descriptions of the obstruction. These include lifting bundle gerbes with non-trivial structure group bundle and bundle gerbes related to the caloron correspondence.

• *A Geometric Model for Odd Differential K-theory* P. Hekmati, M.K. Murray, V. Schlegel and R.F. Vozzo, `arXiv/1309.2834`.

Odd K-theory has the interesting property that it admits an infinite number of nonequivalent differential refinements. In this paper we provide a bundle theoretic model for odd differential K-theory using the caloron correspondence and prove that this refinement is unique up to a unique natural isomorphism. We characterise the odd Chern character and its transgression form in terms of a connection and Higgs field and discuss some applications. Our model can be seen as the odd counterpart to the Simons-Sullivan construction of even differential K-theory.

You can find links to all my publications at `http://www.maths.adelaide.edu.au/michael.murray/publications.html`.

Lie's Third Theorem for infinite-dimensional Lie superalgebras

This text summarizes the main results of my thesis [Ohr15]. Following a construction of Molotkov [Mol84], we can define a supermanifold $\mathcal{M} \in \mathsf{Gr}^{\mathsf{Top}}$ as a functor from the category of finite-dimensional Grassmann algebras Gr to the category of topological spaces, satisfying certain conditions. The underlying picture is the well-known functor of points [DK99] with Gr substituting the opposite category of supermanifolds $\mathsf{SMan}^{\mathrm{op}}$ in the Yoneda lemma. This is possible since Gr turns out to be equivalent to the separating family of *superpoints* $\mathsf{SPoint} \subset \mathsf{SMan}$. The above construction can be seen as a generalization of the classical definition of a supermanifold (see [DK99], for instance) allowing for infinite-dimensional super vector spaces as local models. As usual, a Lie supergroup is then defined as a group object in the category of supermanifolds. In analogy to Lie's Third Theorem for locally convex Lie algebras (see [Nee06] for a very general treatment), we prove the following statement:

Theorem 1
Let $\mathfrak{g} = \mathfrak{g}_0 \oplus \mathfrak{g}_1$ be a locally convex Mackey-complete Lie superalgebra, such that the underlying Lie algebra $\mathfrak{g}$ with $[\mathfrak{g}_1, \mathfrak{g}_1] := 0$ is locally exponential. If the period group $\Pi(\mathfrak{g}_0)$ is discrete, then there is a Lie supergroup $\mathcal{G}$, such that $\mathcal{L}(\mathcal{G}) = \mathfrak{g}$.

There is strong evidence that the requirement for $\Pi(\mathfrak{g})$ to be discrete is also a necessary condition, since this is the case for ordinary Lie algebras. The main proof idea is to reduce the integration of the Lie superalgebra $\mathfrak{g}_1 \oplus \mathfrak{g}_0$ to the integration of the Lie algebra $\mathfrak{g}_0$, which is a well-understood problem in Lie theory ([Nee06], Thm. VI.1.6). To this end, we use the possibility of splitting up a Lie supergroup $\mathcal{G}$ into its underlying Lie group G_0 and a nilpotent factor $\mathcal{G}^{\mathrm{nil}}$ acting on G_0 via a natural transformation $\rho : G_0 \times \mathcal{G}^{\mathrm{nil}} \to \mathcal{G}^{\mathrm{nil}}$ (see [SW11]) and show that even more is true: If the local model of $\mathcal{G}$ is Mackey-complete and G_0 is locally exponential, it is possible to *define* a Lie supergroup as such a triple $(G_0, \mathcal{G}^{\mathrm{nil}}, \rho)$, which we refer to as a *super Harish-Chandra pair*:

Theorem 2
Let $\overline{\mathsf{Grp}(\mathsf{SMan})}$ be the category of Lie supergroups modelled on Mackey-complete locally convex spaces for which the underlying Lie group is locally exponential. Then the category of super Harish-Chandra pairs is equivalent to $\overline{\mathsf{Grp}(\mathsf{SMan})}$.

The above equivalence has not been proven in this generality since most constructions of *supercharts*, serving as a super-analogue of ordinary charts, out of a super Harish-Chandra pair fail to be *supersmooth* (see [Sac07], for instance). Indeed, the construction of supercharts turns out to be the main obstacle in the proof of the above theorem. We trace this back to the problem of endowing an ordinary manifold with a supermanifold structure, which is no longer straight-forward in the infinite-dimensional setting. However, based on the finite-dimensional construction we state a sufficient condition for a manifold to allow such a structure, called *functorizability*. In their book [KM97], Kriegl and Michor treat a very similar problem and show implicitly that functorizability is always fulfilled for the case of manifolds that are modelled on *convenient*, i.e. Mackey-complete locally convex spaces.

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Higher Categories, Localizations and Orbifolds

Dorette Pronk
Dalhousie University

My current research interests are mainly in higher category theory and orbifolds with a view to further develop the interaction between higher category theory and homotopy theory.

Weak Higher Categories. There are several different models for weak higher categories and there are various open questions about how they are related. With Simona Paoli (University of Leicester) I am developing a new model for weak n-categories which based on (strict) n-fold categories with some special properties (with pseudo maps between them). We call these n-fold categories *weakly globular*, because one of the ideas behind this definition is to weaken the notion of globularity (the fact that we have sets of objects, arrows etc.) instead of weakening the notions of units and associativity. We have published our work for the case $n = 2$ [**11**] and are currently working on the details for higher n. The fact that the n-fold categories used are strict themselves makes it easier to work with them, and we are also working on describing the connections between our new model and the existing models.

Besides working on the details of what the higher dimensional structures look like, we have also started developing analogues of various categorical constructions, such as the category of fractions in this context to illustrate how these n-fold categories model weak n-structures [**10**]. Because of the strictness in the weakly globular n-fold categories it will be easier to give the description for the weak n-category of fractions in this context than in the other models that are currently used for weak n-categories.

We hope to be able to use this to further explore the connections between higher category theory and higher homotopy theory.

Localizations. In homotopy theory one often consider the homotopy category obtained by adding inverses to a certain class of arrows (the weak equivalences) subject to some universal property. The simplest way to do this is by adding the reverse arrows and then taking a category of equivalences of zig-zag paths as described by Gabriel and Zisman [**7**]. This has the disadvantage that it is hard to decide whether any two such paths are equivalent and there is no a priori guarantee that the resulting hom-sets will be small. When the class of arrows to be inverted satisfies certain conditions we can take the category of fractions which means that it is sufficient to take paths of length 2 and equality becomes decidable. However, this does not solve any smallness-issues. Quillen model structures are needed to solve those.

In my research I have been considering generalizations of these localizations in which we don't add strict inverses but rather equivalences (this gives the bicategory of fractions [**12**]) or just right adjoints. The latter can be done using the Π_2 construction in [**4**] and subject to some further conditions can also be done using spans [**5**]. The span construction has shown its use in various contexts and Robert Dawson (Saint Mary's University), Bob Paré (Dalhousie University) and I continue to study this construction further in the context of double categories [**3**]. Currently we continue to work out details to place this in the context of oplax/virtual double categories and one of our goals is to use this to explain the relationship between the hammock localization [**6**] and the Π_2 construction [**4**].

Orbifolds. My research on orbifolds and more general, orbispaces is done mostly together with Laura Scull (Fort Lewis College, USA). One of our overarching goals is to understand the commonalities and differences between equivariant homotopy theory and orbifold homotopy theory. We have shown how Bredon cohomology with constant coefficients can be applied to orbifolds. We are currently completing the project on mapping spaces for orbispaces I am speaking about this week. We are also working (with Matteo Tommasini and Alanod Sibih) on a new atlas definition for ineffective orbispaces. From the groupoid point of view it is fairly clear how the effective orbifolds from Satake can be generalized to ineffective orbifolds (where the structure groups of the charts are not required to act effectively). However, finding the corresponding atlas definition requires more subtlety and it turns out that there is more than one way to describe the new atlasses and although they are equivalent, they suggest different notions of morphisms between the resulting orbifolds.

In [**8**] Marco Grandis introduced the manifold construction to describe the local to global process from charts to manifolds. Part of what his construction does is to formalize how one can use partially defined maps in order to be able to describe smooth maps between any two manifolds with their given atlases without needing to change the domain or codomain atlases. This construction was made a bit more transparent by its translation into the language of restriction categories by Robin Cockett and Geoff Cruttwell [**2**]. Together with Cocket I am considering ways to describe orbifolds in the language of restriction categories.

Finally I am quite interested in the orbifold construction for bicategories as used in the study of quantum field theory [**1**], and how this relates to classical orbifolds. I expect that at least one of our new definitions for an orbifold atlas will shine some new light on this question.

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Where does higher geometry come from?

(and other questions)

David Roberts

A lot of my research is aimed at finding examples of higher geometric structures. In particular, the structures that are categorifications of bundles and related data. The theory of such objects has been known rather well for a decade, and the approach via ∞-categories the most encompassing, but this does not supply ready-made examples. This is especially true when one wishes to apply the tools of analysis to higher geometry, where one would like to emulate the successes of gauge theory in ordinary differential geometry.

In my work I've been moderately successful in thinking about the historical origins of the theory of bundles, and what the analogues should be for higher geometry. Natural questions one might ask, and which for me go back to the start of my PhD studies, is whether there is an analogue of the BPST instanton (a certain connection with decay conditions on an $SU(2)$-bundle on $\mathbb{R}^4$) in higher geometry? Is there some sort of action functional one might want to study for which this is an input? Given such a thing, what higher connective structure gives a minimum for this functional? It is not obvious what such an 'action' would be, and even if it should be valued in the usual field of scalars.

I prefer to work with very concrete presentations, though I know that these are merely representatives for objects and maps in a more abstract setting, such as that of ∞-stacks. In particular, I believe that if one can find a *presentation* of a stack, then this information is more useful that knowing just the stack, even if one wishes to prove things in a presentation independent way.

I also believe that higher representation theory will be much intertwined with higher geometry, in particular on higher vector bundles. Exactly what these are is still a little bit mysterious, since one needs to find good analogues of 2-vector spaces (and higher up the ladder).

On a completely different matter, I'm very much interested in how one approaches *class forcing* using categorical techniques. This is a technique, in material set theory, for building new models of ZF(C) from old, where one wants to influence a proper class of sets. One instance is setting the continuum function $\kappa \mapsto 2^\kappa$ at all regular cardinals κ. Forcing using sets is well-known to be equivalent to taking sheaves on a site, for small sites, but class forcing manages to do something similar using *large* sites; in the example above one gets a site as big as the collection of all ordinals. The theory does not extend naïvely from the set forcing case, and so presents an intriguing puzzle, especially as the story is completely unknown if one wants to work in an constructive/intuitionistic setting.

THE GROUP OF HAMILTONIAN HOMEOMORPHISMS AS A TOPOLOGICAL GROUP WITH EVOLUTION OPERATOR

(work in progress)

by Tomasz Rybicki

The group of Hamiltonian diffeomorphisms $Ham(M, \omega)$ of a symplectic manifold (M, ω) consists of the time-one symplectomorphisms of (compactly supported) Hamiltonian isotopies. It is known that $Ham(M, \omega)$ is a Lie group, and that $Ham(M, \omega)$ is simple iff M is compact. Y.G. Oh and S. Müller [OM] introduced a topological Hamiltonian dynamics of a smooth symplectic manifold (M, ω) by means of topological Hamiltonian isotopies. In the definition of topological Hamiltonian isotopy it was used simultaneously both the C^0-convergence on the homeomorphism group of M and the $L^{(1,\infty)}$-convergence on the space of Hamiltonian functions. As a result the group of Hamiltonian homeomorphisms $Hameo(M, \omega)$ of (M, ω) is defined. The whole thing relies on the phenomenon of the C^0-symplectic rigidity discovered by M. Gromov and Y. Eliashberg.

It is interesting that such a C^0-rigidity still holds in the contact category. In view of this observation, recently S. Müller and P. Spaeth in [MS] introduced a topological contact dynamics.

In the paper [BS] L. Buhovsky and S. Seyfaddini established a one-to-one correspondence between topological Hamiltonian isotopies and t-dependent continuous Hamiltonian functions on (M, ω) (by improving an earlier theorem by C. Viterbo). This is a kind of regularity theorem for the group $Hameo(M, \omega)$. Analogous result has been obtained in the contact category. These theorems constitute milestones in the development of C^0-symplectic and C^0-contact dynamics.

Our aim is to look at the group $Hameo(M, \omega$ from the Lie-theoretic point of view (in the wider sense). A very deep and interesting question seems to be whether $Hameo(M, \omega)$ admits a structure of a topological group with Lie algebra (cf. [HM]), and if this depends on the symplectic manifold (M, ω). However, there is another general class of topological groups with some Lie-theoretic features, namely topological groups with evolution operator. This concept exploits the regularity of Lie groups. A rough exposition of the category of topological groups with evolution operator has been presented in [LR]. Concerning $Hameo(M, \omega)$ we stated that this group fulfills the definition of topological groups with evolution operator. Several problems concerning further properties of the group $Hameo(M, \omega)$ arise. Similar facts and questions concern the group of contact homeomorphisms.

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Research Summary

Hadi Salmasian

University of Ottawa, Canada

My current research interests include the following:

1. *Unitary representations of infinite dimensional Lie groups.* This is a project with Karl–Hermann Neeb. In recent papers [4], [5], we have studied analytic aspects of unitary representations of infinite dimensional Lie groups. In [4] we showed that if a Lie group G satisfies the Trotter property, then the space of smooth vectors is equal to the common domain of the unbounded operators coming from the action of the universal enveloping algebra. This result has applications in the representation theory of Lie supergroups (see below) and also Nelson–like characterization of smooth vectors of positive energy representations. This is a forthcoming paper together with Karl–Hermann Neeb and Christoph Zellner.

2. *Spectrum of differential operators on supersymmetric spaces and shifted super jack polynomials.* This is a project with Siddhartha Sahi. In a forthcoming paper we prove that the spectrum of invariant differential operators on the supersymmetric space $GL(m, 2n)/OSp(m, 2n)$ is described by a family of polynomials whose top homogeneous parts are spherical vectors in polynomial representations of $GL(m, 2n)$. We prove that after a suitable change of coordinates to Frobenius coordinates, our polynomials become identical to the shifted super Jack polynomials of Sergeev and Veselov [7]. The latter polynomials describe eigenvalues of "quantum integrals" of the deformed Calogero-Moser-Sutherland system.

3. *Unitary representations of Lie supergroups and C^*-algebras.* The category of Lie supergroups is isomorphic to the category of Harish–Chandra pairs $(G, \mathfrak{g})$, where G is a Lie group and $\mathfrak{g}$ is a Lie superalgebra. Using this correspondence, a notion of unitary representation was defined in [1] for Lie supergroups/Harish–Chandra pairs. In [6], I proved that Kirillov's classical results on the method of orbits extend to nilpotent Lie supergroups. In [3], Karl–Hermann Neeb and myself characterized all simple Lie supergroups which can have nontrivial unitary representations. In a forthcoming paper with Karl–Hermann Neeb, we introduce C^*-algebras for Lie supergroups and prove direct integral decompositions for their unitary representations.

4. *Local and global Weyl modules.* This is a project with Nathan Manning and Erhard Neher. We generalize the theory of local Weyl modules of [2] to root graded Lie algebras, and connect our category of modules to the category of representations of an algebra introduced by Seligman in the study of admissible modules.

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Infinite-dimensional Lie theory, Higher Structures and Application

Alexander Schmeding

My research is inspired by questions from infinite-dimensional Lie theory and its application. Concretely, these questions are connected to three main topics:

1. Lie group structures for groups of diffeomorphisms and differentiable structures on spaces of (differentiable) maps.
2. Connections of infinite-dimensional Lie theory to higher categories, in particular to Lie Groupoids and Lie Algebroids.
3. Infinite-dimensional structures arising in applications in numerical analysis and physics.

Let me briefly illustrate these points with some further comments:

Prime examples of infinite-dimensional Lie groups are the diffeomorphism groups $\mathrm{Diff}(K)$, where K is a smooth and compact manifold. If K is a three dimensional torus, the group $\mathrm{Diff}(K)$ arises naturally in fluid mechanics. The motion of a particle in the fluid corresponds, under periodic boundary conditions, to a curve in $\mathrm{Diff}(K)$.

Moreover, it is possible to construct Lie group structures for diffeomorphism groups of manifolds with "mild singularities", i.e. for diffeomorphisms of orbifolds. However, in studying these infinite-dimensional Lie groups one often discovers natural connections to higher structures in differential geometry, e.g. Lie groupoids and Lie algebroids. Here are two examples for this:

- To deal with morphisms of orbifolds, one should identify orbifolds with (Morita equivalence classes of) certain Lie groupoids. Hence Lie groupoids and their morphisms are naturally connected to the Lie group of orbifold diffeomorphisms.
- To every Lie groupoid G one can associate its group of bisections $\mathrm{Bis}(G)$. Under mild assumptions on G the group $\mathrm{Bis}(G)$ turns out to be an infinite-dimensional Lie group. As a consequence one obtains an interesting connection between the Lie theory of Lie groupoids and Lie algebroids and the Lie theory of infinite-dimensional Lie groups and Lie algebras.

The rich interplay of higher structures and infinite-dimensional Lie theory is one of my research interests.

Finally, I am also interested in infinite-dimensional structures arising in applications in numerical analysis and physics. In numerical analysis infinite-dimensional structures arise naturally in the study of numerical integration schemes and the associated backward-error analysis. For example ,the so called Butcher group is an infinite-dimensional Lie group. It is closely related to a certain Hopf algebra which appears in renormalisation of quantum field theories in physics. These infinite-dimensional structures are often studied from the algebraic point of view to avoid the technical difficulties of infinite-dimensional calculus. My goal is to remedy this situation and bring topology, (infinite-dimensional) analysis and Lie theory back into the picture.

Research Interests

Kay Schwieger (University of Helsinki)

Quantum Markov Process

One of my main research focus are quantum Markov processes. From the point of geometry, these processes typically arise from possibly irreversible actions on non-commutative spaces. My master studies were about generalized Brownian motions introduced by M. Bożejko and R. Speicher. But a maybe more prominent example of quantum Markov processes are convolution processes of (quantum) groups or Lévy processes.

My current particular interest are relations between stochastic processes and the geometry of the underlying space. For example, in recent years D. Goshwami et al. have introduced the quantum isometry group of a non-commutative manifold. I would be eager to learn more about the connection of this quantum group with the differential structure of the manifold.

Interesting problems concern the dilation theory of quantum Markov processes. In a simple form, a dilation can be treated similar to classical coding theory. This interpretation was a key motivation for the coupling method I developed in my Ph. D. thesis. It seems that couplings could also be interesting from the perspective of quantum information theory, but that is maybe another story ...

Beyond mathematical relevance, quantum Markov processes show up in physics as a model of open quantum systems. Currently, I collaborate with a group of experimentalists at Aalto University to refine their model of continuous measurements of a quantum system.

1 Control Theory

In recent years, I have become interested in control theory. In a collaboration with P. Muratore-Ginnaneschi I studied optimal control of the Kramers-Langevin equation, a stochastic differential equation that describes small particles in a noisy environment. For such systems we analyzed and resolved the boundary singularities for minimizing the heat realease.

2 Group Actions on C^*-Algebras

Besides my mainly stochasticly motivated research, I currently collaborate with Stefan Wagner for a project that aims to classify free group actions on general C^*-algebras. If you are interested, you are most welcome to listen to Stefan Wagner's the talk at the workshop about some parts of this project.

Research Interests

Raymond Vozzo

I am interested broadly in bundle gerbes and related structures, as well as the geometry of infinite-dimensional objects such as loop groups and other mapping spaces, and the string group of a compact Lie group.

My main contributions have been surrounding the *caloron correspondence* and its applications. The caloron correspondence was first described as a relationship between instantons on $\mathbb{R}^3 \times S^1$ to monopoles on $\mathbb{R}^3$ with structure group the loop group (these are called calorons). Later it was realised by Murray and Stevenson that it could be restated as a correspondence between and LG-bundles over a manifold M and G-bundles over $M \times S^1$. Together with Murray, I have studied the geometry of this correspondence and used it to calculate characteristic classes of loop group bundles in 'The caloron correspondence and higher string classes for loop groups' (*J. Geom. Phys.*, 60(9) (2010)/arXiv: 0911.3464) and also of certain bundle gerbes in 'Circle actions, central extensions and string structures' (*Int. J. Geom. Methods Mod. Phys.*, 7(6) (2010)/arXiv:1004.0779). Together with Murray and Hekmati in 'The general caloron correspondence' (*J. Geom. Phys.*, **62**(2) (2012)/arXiv:1105.0805) we generalised this correspondence to groups of maps $X \to G$ for X not necessarily equal to S^1. We used this general correspondence in 'The Faddeev-Mickelsson-Shatashvili anomaly and lifting bundle gerbes' (*Comm. Math. Phys.* **319**(2) (2013)/arXiv:1112.1752) with Stevenson to provide an alternative description of the FMS anomaly in quantum field theory using bundle gerbes.

The key point that features in these works, as mentioned above, is that the caloron correspondence can be extended to include the geometry of infinite-dimensional bundles. Recently with Murray, Hekmati and Schlegel we exploited this description of the geometry of loop group bundles to give a bundle-theoretic model for odd differential K-theory, along the same lines as Simon–Sullivan ('A geometric model for odd differential K-theory' To appear in *Diff. Geom. Appl.*/arXiv:1309.2834).

Recently, I have been interested in constructions with groupoids and bundle gerbes. In some work in progress, D. Roberts and I have constructed the groupoid of smooth loops in a Lie groupoid, extending the established constructions of topological loop groupoids (and forming the first step in a program to extend the caloron correspondence to Lie groupoids and objects such as bundle gerbes and orbifolds). We have also constructed explicit examples of string structures using trivialisations of bundle 2-gerbes.

Current research interests

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1. Lie 2-algebras and crossed modules One direction of my research is the interest in *categorified Lie algebras*, i.e. Lie 2-algebras. Crossed modules of Lie algebras constitute an easy way of categorifying Lie algebras, because crossed modules of Lie algebras are in bijection to strict Lie 2-algebras. One can construct representatives $\mu : \mathfrak{h} \to \mathfrak{g}$ of each equivalence class of a crossed module of Lie algebras such that $\mathfrak{h}$ is abelian, which we call "abelian representatives". This leads to many simplifications when working with crossed modules of Lie algebras, for example, this makes it possible in our joint work with H. Abbaspour (Nantes) to define a higher Hochschild cocycle representing the holonomy of a 2-bundle, see `arXiv:1202.2292`.

Furthermore, my thesis student Salim Rivière found an explicit inverse (as a map defined on cochain level) of the antisymmetrisation map

$$HH^*(U\mathfrak{g}, M) \to H^*(\mathfrak{g}, M^{\mathrm{ad}}).$$

This extension of Lie algebra cocycles to Hochschild cocycles is quite complicated and involves a detailed knowledge of $U\mathfrak{g}$ (Eulerian idempotents). The exact translation of crossed modules of $\mathfrak{g}$ into crossed modules of $U\mathfrak{g}$ remains to be explored.

Concerning crossed modules, there are also still many open questions. For example, we have linked explicitely crossed modules of Lie algebras to crossed modules of Hopf algebras. Thus there is the question of how crossed modules of associative algebras (or coassociative coalgebras) are linked to those of Hopf algebras. Another question concerns the construction of 3-cohomology classes (in which cohomology theory ?) associated to crossed modules of Hopf algebras.

The association of a crossed module of Hopf algebras to a crossed module of Lie algebras may be used to define some sort of an enveloping 2-algebra for the string Lie algebra. Which representation-theoretic content has this enveloping 2-algebra ?

2. Leibniz algebras and racks

Recall that a *rack* X is a set with a binary operation $(x, y) \mapsto x \rhd y$ such that for all $x \in X$, the map $x \rhd - : X \to X$ is bijective and for three elements

$x, y, z \in X$, we have the auto-distributivity relation:

$$x \rhd (y \rhd z) \;=\; (x \rhd y) \rhd (x \rhd z).$$

Racks are thus algebraic structures generalizing the conjugation operation in a group.

In work together with B. Dhérin (Berkeley), we transpose a deformation quantization scheme (based on explicit Fourier integral operators involving a generating function for some symplectic micromorphism) which works for the Poisson manifolds given by the duals of Lie algebras to duals of Leibniz algebras. The construction uses the integration of a Leibniz algebra $\mathfrak{h}$ into a Lie rack by the formula

$$X \rhd Y \;:=\; e^{\mathrm{ad}_X}(Y),$$

for all $X, Y \in \mathfrak{h}$. We can then explicitly describe which bracket on the dual of $\mathfrak{h}$ we are quantizing. The properties of this bracket should give a new notion of "generalized Poisson manifold".

In work with C. Alexandre (Strasbourg), M. Bordemann (Mulhouse) and S. Rivière (Nantes), we define a geometric model of the enveloping algebra of a Leibniz algebra using distributions on the local Lie rack associated to the Leibniz algebra. This leads to the notion of a *rack bialgebra*. We end up in this framework with a purely algebraic construction of deformation quantization of Leibniz algebras.

3. Lie rackoids I am thinking with C. Laurent-Gengoux (Metz) on the structure which should integrate Leibniz algebroids. We called this structure Lie rackoids. We have already established some basic properties of these objects.

We have a certain stock of natural examples of rackoids, let me explain one of them to you. Let M be a manifold. Let us define a precategory structure on $\Gamma := T^*M \times M$ by defining for an element $(\alpha, n) \in T^*_m M \times M$ the source map by $s(\alpha, n) = m$, the target map by $t(\alpha, n) = n$ and identities by $\epsilon(m) = (0_m, m)$. One easily computes that the local bisections of this precategory are pairs (ω, ϕ) where ω is a differential 1-form on an open set U and $\phi : V \to U$ is a diffeomorphism. One defines a rack structure (i.e. bisections acting on elements) by

$$(\omega, \phi) \rhd (\alpha, n) \;:=\; (\phi^*\alpha, \phi^{-1}(n)).$$

This gives a natural rackoid structure on the precategory Γ. One computes easily that the infinitesimal Leibniz algebroid corresponding to it is given by $T^*M \oplus TM$ with the Leibniz bracket

$$[(X, \alpha), (Y, \beta)] \;=\; ([X, Y], L_X\beta).$$

It is obvious that this Leibniz algebroid is close to the standard Courant algebroid. While the integration of Courant algebroids has been approached by methods from graded geometry by several research teams, we think about the integration into Lie rackoids. They should have the advantage of being geometrically easier to understand than the constructions involving graded manifolds.

Konrad Waldorf – Research interests

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My research areas are the differential geometry of higher-categorical structures, and mathematical aspects of classical and quantum field theories. I am also interested in the geometry of loop spaces, Lie theory, and homotopy theory.

Central in my research is the notion of a gerbe: a generalization of a fibre bundle over a manifold to a structure whose fibres are categories. Gerbes form a higher-categorical structure und yet allow to study classical differential-geometric aspects such as connections, curvature, and holonomy. Applications of gerbes in the area of 2-dimensional field theories arise because their holonomy can be understood as a coupling between higher-dimensional elementary particles (string) to a gauge field. Their relation to the geometry of loop spaces is established by a procedure called transgression, which transforms higher-categorical geometry over a manifold into ordinary geometry over mapping spaces.

An interesting aspect of the theory of gerbes is that often Lie-theoretical problems arise. This comes essentially from the fact that compact Lie groups carry canonical gerbes, which encode part of the geometry and representation theory of the group. Homotopy theory is relevant because higher-categorical structures can be seen as an instance of infinity-categorical structures, which in turn constitute an algebraic formulation of topology.

Below I outline some of my research interests in more detail. For more details, links, and references, please visit my homepage under

`waldorf.math-inf.uni-greifswald.de`

Multiplicative gerbes and Lie groups

Multiplicative gerbes are gerbes over Lie groups that are compatible with the group structure. They provide a geometric realization of the cohomology of the classifying space of the Lie group. Moreover, connections on multiplicative gerbes provide a geometrical realization of its differential refinement.

In my research I try to extend the general theory of multiplicative gerbes with connections, and I pursue essentially the following two applications. The first application is to Chern-Simons theories; these are 3-dimensional topological field theories of great importance in Mathematics and

Physics. Multiplicative gerbes can help to understand Chern-Simons theories with very general gauge groups, in particular non-simply connected ones. In this context the gerbe generalizes the so-called "level" of the Chern-Simons theory.

The second application is about transgression to the loop group of the underlying Lie group. Under transgression, a multiplicative gerbe with connection becomes a central extension of the loop group. Multiplicative gerbes thus allow a finite-dimensional, higher-categorical perspective on the infinite-dimensional geometry of these central extensions.

String geometry

String geometry is a relatively new research area on the intersection between Algebraic Topology, Differential Geometry, and Homotopy Theory. It provides a mathematical basis for the description of supersymmetry in two-dimensional quantum field theories; from this point of view string geometry is for string theory as spin geometry is for quantum mechanics.

There are essentially two approaches to string geometry: infinite-dimensional analysis on the configuration space of the strings, or higher-categorical analysis on the target space of the strings. The configuration space is the loop space of the target space, and both approaches should be related by a transgression process.

Infinite-dimensional analysis on the loop space leads to long open questions such that how to define a Dirac operator on the loop space, and on which kind of representation this operator could act on. In my work I try to understand these questions via higher-categorical geometry on the target space under transgression.

Transgression to loop spaces

Transgression transforms a gerbe with connection over a manifold M into a line bundle with connection over the free loop space LM, and so establishes a functorial relation between higher-categorical geometry on M and ordinary geometry on LM. In 2-dimensional field theories, for which connections on gerbes represent the gauge fields, the corresponding line bundles play the role of prequantum line bundles, and let us look at the loop space as a kind of symplectic manifold.

For my research the most interesting aspect of transgression is that all line bundles in the image of transgression carry interesting additional structure: so-called fusion products, and equivariance under thin homotopies between loops. These additional structures remember information of the given gerbe that would be lost upon looking at the line bundle alone. Among other things, they admit to invert transgression, and so to go back from infinite-dimensional geometry of LM to higher-categorical geometry over M.

CALCULUS BEYOND MANIFOLDS – JORDAN WATTS
2015/02/05

Introduction. Hamiltonian group actions provide a convenient language for describing classical physical systems using symplectic geometry. For example, they provide tools to study the structure of a Hamiltonian system in a neighbourhood of a periodic flow, and allow one to deduce information about the flow. A couple of methods are readily available, known as symplectic reduction and Poisson reduction, which decrease the number of degrees of freedom of the system, simplifying calculations. However, Poisson reduction requires having a smooth structure on the orbit space of a Lie group action, and symplectic reduction requires a smooth structure on the quotient of a subset of a manifold. None of these quotients are necessarily manifolds themselves, and so what type of smooth structure to equip them with is *a priori* not clear.

One structure that has been developed is that of a *(Sikorski) differential structure*. Basically, this is a sheaf of "smooth" real-valued functions on a set, and sets equipped with such structures form a category which admits subsets and quotients. These have been used by Schwarz and Bierstone to study orbit spaces of compact Lie group actions. Śniatycki uses this theory to extend the Nagano-Sussmann theorem of control theory to locally closed subsets of $\mathbb{R}^n$ and to study reduction of symmetry problems. I currently am writing up a result which shows that there is an essentially injective functor from orbifolds into differential spaces.

A different approach to these singular spaces is the notion of a *diffeology*. A diffeology is a family of "smooth" maps into a set. These structures encode rich information on spaces such as the irrational torus, and have been used to extend Chern-Weil theory from integral closed 2-forms to arbitrary closed 2-forms. They are excellent for working with quotient spaces, but also infinite-dimensional spaces. In our paper [3], Li and I show that the orientation-preserving diffeomorphism group of $\mathbb{S}^2$ has a diffeologically smooth strong deformation retract onto SO(3). This generalises Smale's continuous result, but removes a lot of functional analysis that one might be required to use otherwise.

This idea of using diffeology instead of functional analysis has inspired a few projects in the "infinite-dimensional" realm, including work in progress with Jean-Pierre Magnot in which we are constructing classifying spaces for diffeological groups. Another result is on the space of differential forms of a diffeological space. A result from my PhD thesis [6] shows that the de Rham complex (in the diffeological sense) on an orbit space of a compact Lie group action is isomorphic to the complex of "basic" forms on the original manifold, generalising the known result for compact free Lie group actions. This has since been extended by Karshon and myself [2] to Lie group actions in which the identity component acts properly, in which we show that this isomorphism is also a diffeological diffeomorphism as well. Work in progress with Karshon has the goal of constructing an isomorphism of complexes between diffeological forms on a symplectic quotient with a de Rham complex defined by Sjamaar . Partial results for this appear in my PhD thesis [6]. In a different direction, I show a similar diffeomorphic isomorphism between the complex on the orbit space of a proper Lie groupoid, and the basic forms of the groupoid [7].

I have work in progress with Derek Krepski in which we try to understand geometric prequantisation from the diffeological point of view. However, we have found an obstruction to this: the lack of a theory of vector fields for diffeological spaces. And so a current project of mine is to develop this. The first step is to define a tangent bundle for diffeological spaces, which I have done. For orbit spaces, this matches the so-called stratified tangent bundle, and it matches the classical tangent bundle for path spaces. Another step is to compare this with other definitions by Christensen-Wu and Hector.

Differential structures and diffeology turn out to be "dual" to each other (essentially, one is a family of maps into a set, whereas the other is a family of maps out of it). This is studied, with many examples, in my paper with Batubenge, Iglesias-Zemmour, and Karshon [1]. This is important for understanding how to generalise calculus from manifolds, and how various spaces (*e.g.* singular varieties, orbit spaces, infinite dimensional spaces, etc.) relate to one another. However, both of these structures ignore isotropy. Indeed, while diffeology often remembers information about an equivalence relation on a set, it cannot tell the difference between two Lie group actions (for example) if they induce the same equivalence relation. To capture the missing isotropy information, one must go to Lie groupoids and stacks. Wolbert and I in [8] show that a stack has an "underlying diffeological structure", only depending on the isomorphism class of the stack, and in the case of a stack representing a Lie group action, this diffeology is exactly the quotient diffeology on the orbit space. Hence we have developed a functor that forgets the isotropy information, leaving behind the information gained from the equivalence relation alone.

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Research Interests

Christoph Wockel

My research interests roughly divide into three areas:

Topological group cohomology

Topological group cohomology is the cohomology theory of topological and Lie groups that captures simultaneously the algebraic *and* topological properties of a given such group. At the moment, I am working on establishing links of topological group cohomology with the following closely related subjects:

- Chern-Weil Theory of compact symmetric spaces
- Lie groupoid cohomology
- bounded continuous (and smooth) cohomology

Lie groupoids and their bisections

(with Alexander Schmeding) Lie groupoids and infinite-dimensional Lie groups are both generalisations of finite-dimensional Lie groups. However, these two subjects have in the past been studied mostly simultaneously. The aim of my work in this direction is to establish a close link between these two subjects by the functor that assigns to each Lie groupoid its (Lie) group of bisections. One crucial observation here is that one can view each Lie groupoid as a quotient of an action groupoid for the natural action of the bisections on the objects. Interesting questions and applications of this perspective should arise from

- the cohomology of Lie groupoids and infinite-dimensional Lie groups,
- integrability questions of Lie algebroids and their Lie algebras of sections and
- implications of the relation of the integrability obstructions on low-dimensional homotopy groups of diffeomorphism groups.

String Geometry

(in parts with Christian Becker, Michael Murray and David Roberts) One perspective to string geometry is that it is the vertical categorification of spin geometry. Categorification amounts here to a shift in the cohomological dimension, so it is consistent with many observations and constructions in string theory. Questions that I am particularly interested in this subject are the following.

- constructing, understanding and classifying models for the string 2-group
- representations of Lie 2-groups, and in particular of the string 2-group
- geometric string structures and string connections

Research Summary

Chenchang Zhu

Higher categorical structures attract much attention in various areas in mathematics. My current research focuses on *higher categorical structures in differential geometry*. In a certain sense, my projects can be viewed as providing "categorification" of various classical problems. The categorified results have the advantage of being either conceptually complete (giving solutions to unsolvable problems which were unsolvable beforehand), or structurally richer (discovering new properties).

More concretely, I'm interested in

(1) higher Lie theory studying infinitesimal and global symmetries.
(2) higher categories of higher groupoids.
(3) Courant algebroids.